Advanced MOSFET Modeling

Slide 1

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The slide illustrates silicon crystal lattice.

Potential minima for electrons are formed near the atom nuclei.

One free electron is shown.

Between the valence and the conduction band there is a band gap.

Slide 2

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Concept of quantum states

Electrons are Fermi-particles – Fermions

Only one electron can exists in one quantum state

Quantum states are illustrated with boxes around atoms, or with lines in the energy diagram

Slide 3

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In the valence band, nearly all quantum states are filled with electrons

In the conduction band, very few states are filled

Slide4

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Intrinsic semiconductor

Slide 5

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Acceptor atom leads to an energy state that is a little bit above the valence band. Acceptor is neutral when empty. Slide 5 shows an empty acceptor state. This would be so at very low temperature.

Slide 6

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Slide shows the acceptor state filled with electron. One electron jumped from the top of the valence band to the acceptor state. The acceptor atom is therefore ionized and negative.

Notice that one state in valence band is now empty. This empty state can now “move” though the valence band and in this way transports positive charge – “hole”.

Slide 7

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Slide 7 shows a donor atom. This atom has a state for an electron that is a little below the conductive band. Donor is neutral when filled with an electron. Slide shows the filled donor state – it would be so at a very low temperature.

Slide 8

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At a little higher temperature the electron can jump from the donor state to the conductive band. The donor is now ionized and positive. Electron in conductive band is “free” and can contribute to current.

Slide 9

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This slide shows three distribution functions.

$$n = n_0 e^{-\frac{E}{kT}}$$

is the Maxwell-Boltzmann distribution. It can be derived for the particles that are distinguishable (like the balls with numbers) and unlimited number of them can occupy one quantum state. It holds for classical particles.

$$n = \frac{1}{e^{\frac{E-\mu}{kT}} - 1}$$

is the Bose-Einstein distribution. It can be derived for the particles that are not distinguishable and unlimited number of them can occupy one quantum state. It holds for Bosons (like photon).

$$n = \frac{1}{e^{\frac{E-E_f}{kT}} + 1}$$

is the Fermi-Dirac distribution. It can be derived for the particles that are not distinguishable and only one can occupy one quantum state. It holds for Fermions, such as the electron. Fermi-Dirac distribution has a parameter called Fermi-Energy (Ef). We can think about is as a statistical parameter like temperature. Temperature is the measure of the average energy and Ef is the measure of the number of particles. At T=0K, all the quantum states with energies E<Ef are filled and all E>Ef are empty.

Slide 10

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This slide illustrates the Maxwell-Boltzmann distribution. The density of particles (or probability that a state is full) is given by:

$$n = n_0 e^{-\frac{E}{kT}}$$

E is the potential energy. For instance, we can use this formula to calculate the air density versus height. In this case, E is the gravitational potential energy E = mgh.

Slide 11

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This slide illustrates the Fermi-Dirac (FD-) distribution. This distribution has a very different shape than the Maxwell-Boltzmann (MB) distribution only around the Fermi energy. For energies well above the Fermi energy (many kT above) the FD distribution can be approximated by MD distribution.

$$n = e^{-\frac{E-E_f}{kT}} = n_0 e^{-\frac{E_e}{kT}}$$

For energies much below the Ef (many kT below), the number of **empty** states follows the formula similar to the MB distribution

$$1 - n = e^{-\frac{E_f-E}{kT}} = p_0 e^{\frac{-E_h}{kT}}$$

Therefore: If there is an energy gap around the Fermi energy, the number of electrons in the conduction band and the number of holes in the valence band can be approximated by Maxwell-Boltzmann distributions.

Slide 12

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The side gives formulas that we will use later.

C is capacitance, ψ is electrostatic potential, Q is charge, E electric field.

Red is positive charge, blue is negative charge.

The capacitance of the capacitor does not depend on the thickness of the electrodes.

Slide 13

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The slide shows the n-channel MOSFET structure. Upper part shows the cross section. The figure shows source and drain (n-regions), and the gate electrode above.

The lower part of the slide shows the energy diagram (conduction and valence band) that corresponds to the dashed line. Blue regions represent free electrons. Yellow regions represent holes. Red bricks illustrate positive depleted donors. Blue bricks illustrate negative depleted acceptors.

We have two PN-junctions. The regions, where potential is changing, are depleted. The regions with low potential in conduction band contain free electrons (blue). The regions with high potential in the valence band contain holes (yellow).

Slide 14

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Notice that the charge of the donors and acceptors in the depleted layer is not compensated. This charge makes electric field (arrows) and causes electric potential change ψ. The field direction is from the positive to the negative charge (arrow). The electric potential ψ decreases in the direction where electric field is pointing. The energy diagram shows the potential energy for electrons that is equal to

Ep = -e ψ

Therefore the potential energy has the opposite sign from the electric potential.

Slide 15

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As explained in slide 11, the densities of electrons and holes can be calculated using MB distribution functions because the FD distribution, that holds exactly, can be approximated in this way.

Let us calculate the density of electrons in the p-region below the gate. The electrons are minority carriers there.

We start from the n-regions – source and drain. The electron density in source / drain is equal to Nd – the density of donors. If the potential increase from n to p region is E = e Vbi, the density of electrons in the p-region is:

$n_p = N_d e^{-\frac{E}{kT}}$ (1)

In analogous way, we can calculate the density of holes in the source and drain:

$p_n = N_a e^{-\frac{E}{kT}}$ (2)

Notice that for every position holds:

$$n \times p = const = N_a \times N_d \times e^{-\frac{E}{kT}}$$

Slide 16

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In order to derive the value for $n \times p$, let us imagine that there is an intrinsic (non-doped) silicon part attached to the drain. In this part the electron- and hole densities are equal and they are equal to parameter ni.

Since $n \times p$ is constant everywhere and $n = ni$ and $p = ni$ in the intrinsic part, it holds:

$n \times p = n_i^2$ (3)

Ni is equal to

$$n_i = 1.45 \times 10^{10} cm^{-3}$$

Notice that typical doping is six orders of magnitude smaller than the atom density, and the intrinsic charge density is six orders of magnitude smaller than the typical doping in silicon.

Using equation (3), we can calculate the minority carrier densities in an alternative way (compare with (1) and (2)):

$$n_{p0} = \frac{n_i^2}{N_a} \quad p_{n0} = \frac{n_i^2}{N_d}$$

Slides 17 - 19

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The slides illustrate how the band diagram and the densities of electrons and holes change, when we apply positive voltage at the gate electrode.

Slide20

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The slide shows 2D potential diagram of the valence band. The z axis is energy and the y-axis the depth of the MOS structure.

Slide21

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The slide shows 2D potential diagram of the conduction band.

Slide 22

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Let us now calculate the amount of charge in the channel when we apply a gate-bulk voltage Vgb.

The region of the MOSFET within the dotted square will be examined in the following slides.

We distinguish the voltage in gate oxide Vox, and the potential change in the depleted region ψs.

It holds: Vgb = Vox + ψs.

An important initial conditions for the calculations are the hole- and electron densities are

$$p_p = N_a$$
$$n_p = n_i^2 / N_a \quad (4)$$

These are the densities at the bottom edge of the depleted region.

Slide 23

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Slide 23 shows magnified the region within the square from the slide 22.

We would like to calculate the channel charge as function of the gate-bulk voltage

$Q'_{ch}(V_{GB}) = ?$

It holds

$V_{GB} = \psi_s + V_{ox}$ (5)

The positive charge at the gate Qg, is equal to the negative charge in the channel Qch and the negative charge due to the acceptors in the depleted region Qdep.

$Q'_G = Q'_{ch} + Q'_{dep}$ (6)

Only in this way, the structure is electroneutral.

The voltage drop in the oxide can be calculated using the formula for the plate capacitor

$V_{ox} = \frac{Q'_G}{C'_{ox}} = \frac{Q'_{ch} + Q'_{dep}}{C'_{ox}}$ (7)

$C'_{ox} = \frac{\varepsilon_{SiO_2}\varepsilon_o}{t_{ox}}$ (8)

We used equation (6) to derive (7).

If we calculate the Qch and Qdep as functions of ψs, using eq (7) and (5) we can derive Vgb as function of ψs. Using (6) we can then find Qch (Vgb). Therefore our first tasks are to calculate

$Q'_{ch}(\psi_s) = ?$

and

$Q'_{dep}(\psi_s) = ?$

Slide 24

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Let us a bit simplify the MOSFET structure by setting the oxide thickness to 0. In this case Vox = 0, and Vgb = ψs. Except of this nothing else changes.

Slide 25

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Let us calculate the acceptor charge in the depleted layer as function of voltage ψs.

$Q'_{dep}(\psi_s) = ?$

We will first define the dynamic capacitance of the depleted layer/unit area C'. This capacitance is defined as

dQ/dψ

dQ is the change of the gate charge (and the depleted region charge) and dψ is the corresponding change of potential ψs.

The middle figure shows the existing charge Q, the additional charges (at the gate and in the depleted region) dQ, as well as ψs and dψ. The right figure illustrates that dψ is actually caused by dQ. The additional charges are separated by the original charge Q. If the size of the depleted region is t, the dynamic capacitance dQ/dψ is given by the formula for a plate capacitor:

$$\frac{dQ'}{d\psi} = C' = \frac{\varepsilon_r \varepsilon_0}{t} \quad (9)$$

Let us now multiply the denominator and numerator of last equation with eNa (e – elementary charge, Na is the acceptor density).

Since the charge in the depleted region is given by

$$Q' = eN_a t \quad (10)$$

It holds

$$\frac{dQ'}{d\psi} = \frac{\varepsilon_r \varepsilon_0 e N_a}{Q'} \quad (11)$$

Equation (11) is a differential equation that allows calculation of Q(ψs).

Let us first start from (11) and separate the variables

$$Q' dQ' = \varepsilon_r \varepsilon_0 e N_a d\psi$$

By integration of both sides we obtain:

$$\frac{1}{2} Q'^2 = \varepsilon_r \varepsilon_0 e N_a \psi_s$$

Or

$$Q' = \sqrt{2 \varepsilon_r \varepsilon_0 e N_a \psi_s} \quad (12)$$

Equations (10) and (12) lead to

$$t = \sqrt{\frac{2\varepsilon_r \varepsilon_0 \psi_s}{eN_a}} \quad (13)$$

Finally (9) and (12) lead to

$$C^{'} = \sqrt{\frac{\varepsilon_r \varepsilon_0 e N_a}{2\psi_s}} \quad (14)$$

Slide 26

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Let us now calculate the charge in the channel as function of ψs.

Notice, that the channel is built of free electrons. The free electrons follow the MB-distribution – there are more and more electrons when we approach the gate oxide because the electron potential energy decreases in this direction. As consequence, the channel will occupy only a thin region near the gate oxide. (Notice that, for simplicity, we assumed that oxide thickness is zero.)

The channel charge will be calculated in the next slide. In this slide, we derive the change of the potential ψ in the thin region dy near the gate oxide.

The potential d ψ is caused by the charge Q. It holds

dψ = Q/C(dy)

C(dy) is here the capacitance of the small piece of silicon with thickness dy.

It holds:

$$d\psi = \frac{Q^{'}}{C^{'}(dy)} = \frac{eN_a t}{\frac{\varepsilon_r \varepsilon_0}{dy}} = \frac{eN_a dy}{\frac{\varepsilon_r \varepsilon_0}{t}} = \frac{eN_a dy}{C^{'}} \quad (15)$$

where we used result (9).

C’ is the capacitance of the depleted region (14).

Slide 27

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Let us now calculate the charge in the channel as function of ψs.

At first, it holds:

$$n_p = e n_{p0} e^{\frac{\psi(y)}{U_T}} \quad (16)$$

np is the density of the electrons in the depleted layer. Np0 is the electron density at the bottom edge of the depleted layer where it holds:

$$\begin{aligned} p_p &= N_a \\ n_p &= n_i^2 / N_a \equiv n_{p0} \end{aligned} \quad (17)$$

UT is the thermal voltage (kT/e), UT ~ 25mV at room temperature.

Let us now linearize the potential ψ close to the upper edge of the deplete region (the region where channel is formed).

$\psi(y) = \psi_s + \frac{d\psi}{dy} y$ (18)

From equation (15) we obtain

$\frac{d\psi}{dy} = -\frac{eN_a}{C^{'}}$ (19)

By substituting (19) into (18), and (18) into (16), we get:

$n_p = en_{p0} e^{\frac{\psi_s}{U_T}} e^{-\frac{eN_a y}{C^{'} U_T}}$ (20)

The total channel charge is obtained by integration of equation (20)

$Q^{'}_{ch} = \int_0^t en_{p0} e^{\frac{\psi_s}{U_T}} e^{-\frac{eN_a y}{C^{'} U_T}} dy$

The result is:

$Q^{'}_{ch} = \frac{C^{'} U_T}{eN_a} en_{p0} e^{\frac{\psi_s}{U_T}}$ (21)

C’ is the capacitance of depleted region (14).

Slide 28

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Now we have all equations and formulas and we can calculate numerically Qch(Vgb).

Let us summarize.

Equation 21 gives the channel charge as function of depletion layer potential ψs.

$Q^{'}_{ch} = C^{'}(\psi_s) U_T \frac{n_i^2}{N_a^2} e^{\frac{\psi_s}{U_T}}$ (22)

Equation 12 gives the depletion layer chare charge as function of depletion layer potential ψs.

$Q^{'}_{dep} = \sqrt{2\varepsilon_r \varepsilon_0 e N_a \psi_s}$ (23)

It holds from (5,6,7):

$V_{GB} = \psi_s + V_{ox} = \psi_s + \frac{Q'_{ch}(\psi_s) + Q'_{dep}(\psi_s)}{C'_{ox}}$ (24)

The functions Qch(Vgb) and ψs (Vgb) are shown. Notice that Qch is quite small until ψs reaches the value 2ψ0. After this, ψs saturates and Qch increases linearly.

The constant ψ0 is defined (calculated) as:

$\psi_0 \equiv U_T \ln \frac{N_a}{n_i} = 0.42V$ (25)

Using MB-distribution (16), it can be derived that when ψs = 2ψ0, the electron density at the upper edge of the depletion region is equal to Na. Therefore the electron density is equal to the hole-density in the undepleted p-silicon. We are talking about inversion.

Slide 29

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In following slides we will approximate the equation (24) for the regions of strong- and weak inversion.

The region of strong inversion is highlighted with grey. In this region, the potential ψs is nearly constant and equal to

$$\psi_s \approx 2\psi_0 = 2U_T \ln \frac{N_a}{n_i}$$

Some books use the value $2\psi_0 + 6U_T$ as onset of strong inversion.

The equation (24) can be then simplified as follows

$V_{GB} = 2\psi_0 + \frac{Q'_{ch} + Q'_{dep}(2\psi_0)}{C'_{ox}}$ (26)

Let us now define the parameter **threshold voltage** as the Vgb voltage where the strong inversion starts (ψs reaches the value 2 ψ0). It holds:

$V_{TH} = V_{GB}(2\psi_0) = 2\psi_0 + \frac{Q'_{dep}(2\psi_0)}{C'_{ox}}$ (27)

By substituting (27) into (26) we obtain:

$Q'_{ch} = C'_{ox}(V_{GB} - V_{TH})$ (28)

This is the channel charge equation in strong inversion.

Slide 30

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The region of weak inversion is highlighted with grey. In this region, the potential ψs changes nearly linearly as function of Vgb, and the channel charge is much smaller than the depletion region charge.

Equation (24) can be then simplified as follows:

$$V_{GB}(\psi_s) = \psi_s + \frac{Q'_{dep}(\psi_s)}{C'_{ox}} \quad (29)$$

We would like to express ψs as function of Vgb. In order to do this, we first linearize the equation (29) in the vicinity of 2 ψ0.

$$V_{GB} = V_{GB}(2\psi_0) + \left.\frac{dV_{GB}}{d\psi_s}\right|_{2\psi_0} (2\psi_0 - \psi_s)$$

If we take (29) into account, it holds:

$$\left.\frac{dV_{GB}}{d\psi_s}\right|_{2\psi_0} = 1 + \frac{1}{C'_{ox}} \left.\frac{dQ'_{dep}}{d\psi_s}\right|_{2\psi_0}$$

Further:

$$C' = \frac{dQ_{dep}}{d\psi_s}$$

Therefore:

$$V_{GB} = V_{GB}(2\psi_0) + \left(1 + \frac{C'(2\psi_0)}{C'_{ox}}\right)(2\psi_0 - \psi_s)$$

If we use the definition of the threshold voltage (27), we can rewrite the last equation as:

$$\psi_s = 2\psi_0 + \frac{V_{GB} - V_{TH}}{n} \quad (30)$$

The slope factor n is defined as:

$$1 + \frac{C'(2\psi_0)}{C'_{ox}} \equiv n \quad (31)$$

Equation (21) gives us the channel charge as function of ψs:

$$Q'_{ch} = C'(\psi_s) U_T \frac{n_i^2}{N_a^2} e^{\frac{\psi_s}{U_T}} \quad (32)$$

If we substitute equation (30) in (32), we obtain the approximate equation for the channel charge in weak inversion:

$$Q'_{ch} \approx (n-1)C'_{ox}U_T e^{\frac{V_{GB}-V_{TH}}{nU_T}} \quad (33)$$

Notice that we replaced C(ψs) with C(2 ψ0).

Slide 31

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Now we have all important equations:

Channel charge versus gate-bulk voltage in weak inversion (33), channel charge versus gate-bulk voltage in strong inversion (28), the definition of the threshold voltage (27), the definition of the ψ0 (25), definition of the slope factor (31), the depleted layer capacitance (14) and the depleted layer charge (12).

We have derived these formulas under assumption that the density of minority carriers (electrons) at the lower edge of the depleted region is

$$n_p = n_i^2 / N_a$$

Notice that the term ni^2/Na is a part of the equation for ψ0 (25).

$$\psi_0 \equiv U_T \ln \frac{N_a}{n_i} = U_T \ln n_i \frac{N_a}{n_i^2}$$

Also the threshold voltage depends indirectly on ψ0 since Vth depends on ψ0.

If the density of minority carries changes, the formulas for Vth and ψ0 would need to be adjusted.

This is the case when we have the drain at a nonzero potential versus the bulk – when we have the drain bias.

Slides 32 and 33

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The slides illustrate what happens in the case of the drain bias Vdb. The drain-bulk junction is now reversely polarized and the potential change in the depleted region near drain is increased by Vdb. The size of the depleted region is increased as well.

It can be derived that the electron density at the bottom edge of the depleted region, close to drain, is:

$$n_p = \frac{n_i^2}{N_a} e^{-\frac{V_{DB}}{U_T}} \quad (34)$$

This can be derived starting from MB-distribution for electrons (1) applied to the “paths” A and B, which are nearly in thermodynamic equilibrium and from the fact that the density of electrons in the drain and the source is Nd. The density in the point PB must be by factor $e^{-\frac{V_{DB}}{U_T}}$ smaller than the density in point PA. Since the density in PA is $n_p = \frac{n_i^2}{N_a}$ in PB must be $n_p = \frac{n_i^2}{N_a} e^{-\frac{V_{DB}}{U_T}}$.

The question mark in slide 32 illustrates that the semiconductor is not in the thermodynamic equilibrium along the entire depletion region boundary. The MB-distribution does not hold everywhere.

Slide 34

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This slide introduces the corrections for the formulas close to the drain, due to the drain bias.

Following the result (34), the formula for ψ0 should be corrected as:

$$2\psi_0 \rightarrow 2\psi_0 + V_{DB} \quad (35)$$

Let us replace this in the formula for threshold voltage (27). We obtain:

$$V_{TH} -> 2\psi_0 + V_{DB} + \frac{Q'_{dep}(2\psi_0 + V_{DB})}{C'_{ox}} = 2\psi_0 + \frac{Q'_{dep}(2\psi_0)}{C'_{ox}} + V_{DB} + \left.\frac{dQ'_{dep}}{C'_{ox} d\psi_s}\right|_{2\psi_0} V_{DB}$$

$$= V_{TH} + V_{DB}(1 + \frac{C_{dep}(2\psi_0)}{C'_{ox}})V_{DB} = V_{TH} + nV_{DB} \quad (36)$$

By substituting (35) and (36) into (28) and (33) we obtain

$$Q'_{ch} \approx C'_{ox}\left(V_{GB} - V_{TH} - nV_{DB}\right) \quad (37)$$

for strong inversion and

$$Q'_{ch} \approx (n-1)C'_{ox}U_T e^{\frac{V_{GB} - V_{TH}}{nU_T}} e^{-\frac{V_{DB}}{U_T}} \quad (38)$$

for weak inversion.

Slide 35

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In the case of source-bias, similar correction by Vsb can be performed for the channel charge equation close to the source.

Slide 36

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If we want to calculate the channel charge in any point of the channel between the source and drain, we can start from the equations (37) and (38). There are two approaches.

1. Bulk is the reference

Within the first approach, the bulk voltage is the reference. We use the formulas (37) and (38), and replace Vdb with Vx. Vx changes from Vsb (channel close to source) to Vdb (channel close to drain). The threshold is defined as in (27).

The equations are:

$$Q'_{ch} \approx C'_{ox}\left(V_{GB} - V_{TH0} - nV_x\right) \quad (39)$$

$$Q'_{ch} \approx (n-1)C'_{ox}U_T e^{\frac{V_{GB}-V_{TH0}}{nU_T}} e^{-\frac{V_x}{U_T}} \quad (40)$$

$$V_x \in (V_{SB}, V_{DB})$$

The threshold is defined as

$$V_{TH0} = 2\psi_0 + \frac{\sqrt{2\varepsilon_r\varepsilon_0 e N_a\left(2\psi_0\right)}}{C'_{ox}} \quad (41)$$

2. Source is the reference

Another possibility is to take the source as the reference. Near the source, we do following transformations: (compare with 36)

$$V_{TH} -> 2\psi_0 + V_{SB} + \frac{Q'_{dep}(2\psi_0 + V_{SB})}{C'_{ox}} \quad (42)$$

We redefine the threshold as:

$$V_{TH} = 2\psi_0 + \frac{Q'_{dep}\left(2\psi_0 + V_{SB}\right)}{C'_{ox}} = 2\psi_0 + \frac{\sqrt{2\varepsilon_r\varepsilon_0 e N_a\left(2\psi_0 + V_{SB}\right)}}{C'_{ox}} \quad (43)$$

Therefore

$$V_{TH} -> V_{TH} + V_{SB} \quad (44)$$

By substituting the transformation (44) into (28) we obtain

$$Q'_{ch} \approx C'_{ox}\left(V_{GS} - V_{TH}\right) \quad (45)$$

By substituting the transformation (44) into (33) we obtain

$$Q'_{ch} \approx (n-1)C'_{ox}U_T e^{\frac{V_{GS}-V_{TH}}{nU_T}} \quad (46)$$

The slope factor is defined as:

$$1+\frac{C^{'}(2\psi_0+V_{SB})}{C^{'}_{ox}}\equiv n \quad (47)$$

In an arbitrary point in the channel we have:

$$Q^{'}_{ch}\approx C^{'}_{ox}\left(V_{GS}-V_{TH}-nV_x\right) \quad (48)$$

$$Q^{'}_{ch}\approx (n-1)C^{'}_{ox}U_T e^{\frac{V_{GS}-V_{TH}}{nU_T}} e^{-\frac{V_x}{U_T}} \quad (49)$$

$$V_x\in(0,V_{DS})$$

Slides 37 – 44

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These slides just summarize the results we have obtained so far.

Slide 45

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Let us now derive the source-drain current equation for strong inversion. We will assume source as reference and use the channel-charge equation (48). We assume that in strong inversion, the drift current dominates. The drift current equation is:

$$I(x)=W\mu Q^{'}_{ch}\frac{dV_x}{dx}=const(x) \quad (50)$$

Factor μ is mobility, W is gate width, dVx/dx is the electric field. Notice that although Q and electric field are position dependent, the current is constant through the channel.

Equation (50) can be integrated

$$\int I dx=\int W\mu Q^{'}_{ch}\frac{dV_x}{dx}dx$$

$$\int_0^L I dx=\int_0^{V_{DS}} W\mu Q^{'}_{ch} dV_x$$

$$IL = \int_0^{V_{DS}} W\mu Q'_{ch} dV_x$$

Finally, we obtain the current in the form of the integral:

$$I = \mu \frac{W}{L} \int_0^{V_{DS}} Q'_{ch} dV_x \quad (51)$$

The next step would be to substitute (48) into (51) and perform integration.

Slide 46

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Let us derive the source-drain current equation, now, for weak inversion. We will assume source as reference and use the channel-charge equation (49). We assume that in weak inversion, the diffusion current dominates. The diffusion current equation is:

$$I(x) = -WD\frac{dQ'_{ch}}{dx} = const(x)$$

D is the diffusion constant.

Also here, we can perform the integration

$$\int I(x)dx = -\int WD\frac{dQ'_{ch}}{dx}dx$$

$$\int_0^L Idx = -\int_0^{V_{DS}} WD\frac{dQ'_{ch}}{dV_x}dV_x$$

If we now substitute the channel charge equation (49) into last equation we obtain:

$$I = \mu \frac{W}{L} \int_0^{V_{DS}} Q'_{ch} dV_x \quad (52)$$

It is very interesting that this equation has the same form as the equation for the current in strong inversion (51). This means than the same formula (51 and 52) can be used independently of the working mode of the transistor. The only difference for strong- and weak inversion is the equation for Qch.

Slide 47

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This slide shows the current equations for weak- and for strong inversion, obtained after integration of formula (51).

The current in weak inversion is

$$I = \mu C'_{ox} \frac{W}{L}(n-1)U_T{}^2 e^{\frac{V_{GST}}{nU_T}}\left(1-e^{-\frac{V_{DS}}{U_T}}\right) \text{ (53)}$$

The current in strong inversion is (source is reference)

$$I = \mu C'_{ox} \frac{W}{L}\left(V_{GST}V_{DS} - n\frac{V_{DS}{}^2}{2}\right) \text{ (54)}$$

If bulk is used as reference, the current is (here (39) is used instead of (48))

$$I = \mu C'_{ox} \frac{W}{L}\left[(V_{GBT} - nV_{SB})^2 - (V_{GBT} - nV_{DB})^2\right] \text{ (55)}$$

with

$$V_{GST} \equiv V_{GS} - V_T;\ V_{GBT} \equiv V_{GB} - V_{T0} \quad \text{(56)}$$

Slide 48

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This slide shows the current saturation in the case of weak inversion. The saturation occurs when Vds reaches several thermal voltages n x UT.

Slide 49

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This slide shows the current saturation in the case of strong inversion. Formulas for source as reference are used. The saturation occurs when Vds reaches the value (called Vdssat)

$$V_{DSSAT} = \frac{V_{GS} - V_{TH}}{n} \text{ (57)}$$

The saturation current is

$$I_{sat} = \mu C'_{ox} \frac{W}{L}\frac{V_{GST}{}^2}{2n} \text{ (58)}$$

Slide 50

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This slide shows the current saturation in the case of strong inversion. Formulas for bulk as reference are used.

The drain current equation is (55)

$$I = \mu C'_{ox} \frac{W}{L} \left[(V_{GBT} - nV_{SB})^2 - (V_{GBT} - nV_{DB})^2 \right]$$

Notice that the equation has two separated terms, one with Vsb and one with Vdb. This gives us the idea to represent the transistor as a parallel connection of two devices that produce currents:

$I = \mu C'_{ox} \frac{W}{L} (V_{GBT} - nV_{SB})^2$ and $I = \mu C'_{ox} \frac{W}{L} (V_{GBT} - nV_{DB})^2$ respectively. This is one of the ideas in the EKV transistor model developed at EPFL.

Slide 51

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The Berkeley transistor model (BSIM) takes into account one additional effect – the mobility saturation.

The current equations is the same as (50)

$$I(x) = W\mu Q'_{ch} \frac{dV_x}{dx} = const(x) \quad (59)$$

However, the mobility is given by the equation:

$\mu = \frac{\mu_0}{1 + E_x / E_{sat}}$ (60) for electric field lower than some saturation value Esat.

For higher field values, the current does not depend anymore on the field and is given by the equation:

$$I(x) = WQ'_{ch} v_{sat} = WQ'_{ch} \frac{\mu_0}{2} E_{sat} \quad (61)$$

This formula can be derived from (59) and (60) by replacing Ex with Esat.

The integration of (59) with (60) leads to:

$$\int I \left(1 + \frac{1}{E_{sat}} \frac{dV_x}{dx} \right) dx = \int W \mu_0 Q'_{ch} \frac{dV_x}{dx} dx$$

and finally:

$$I = \frac{\mu_0 C'_{ox} W}{L\left(1 + \frac{V_{DS}}{LE_{sat}}\right)} \left(V_{GST} - n\frac{V_{DS}^{\ 2}}{2}\right) \ (62)$$

This equation is similar as (54) but it has the correction factor: $1 + \frac{V_{DS}}{LE_{sat}}$.

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Let us now calculate the saturation current.

Within BSIM model, the current saturation is explained with velocity saturation. The current saturates when at the drain side, the electric field in the channel achieves the value Esat. Then the current is given by the equation

$$I_{sat} = v_{sat} W Q'_{ch}(L) \ (63)$$

The charge in the channel is given by (48):

$Q'_{ch} \approx C'_{ox}(V_{GST} - nV_x)$, with $V_{GST} \equiv V_{GS} - V_T$ (64)

Let us first derive the drain-source voltage, for which the saturation occurs Vdssat.

From (63) and (64) we can write

$$I_{sat} = \frac{\mu_0}{2} E_{sat} W C'_{ox}(V_{GST} - nV_{DSSAT}) \ (65)$$

It holds also (62) at the onset of saturation. Therefore:

$$\frac{\mu_0 C'_{ox} W}{L\left(1 + \frac{V_{DSSAT}}{LE_{sat}}\right)} \left(V_{GST} - n\frac{V_{DSSAT}^{\ 2}}{2}\right) = \frac{\mu_0}{2} E_{sat} C'_{ox} W (V_{GST} - nV_{DSSAT}) \ (66)$$

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By solving equation (66), Vdssat can be obtained:

$$V_{DSSAT} = \frac{V_{GST}}{n\left(1 + \frac{V_{GST}}{nLE_{sat}}\right)} \ (67)$$

Compare this with equation (57). The BSIM model introduces the correction: $\left(1+\frac{V_{GST}}{nLE_{sat}}\right)$.

By substituting (67) into (62) we obtain the saturation current:

$$I_{sat} = \frac{\mu_0 C'_{ox} W}{L\left(1+\frac{V_{GST}}{nLE_{sat}}\right)} \frac{V_{GST}^2}{2n} \quad (68)$$

Compare this with (58). Typical values for the transistor parameters are shown in the slide. Notice that for small L and large Vgst, the factor $\left(\frac{V_{GST}}{nLE_{sat}}\right)$ may be larger than 1. In this case, the current increases only linearly with Vgst.

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This can influence the trans-conductance as well, as shown in this slide.

The trans-conductance reaches the maximal value $v_{sat} W C'_{ox}$ and does not grow anymore when Vgst is increased. In weak inversion, the trans-conductance is linearly proportional to the current.

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It is interesting that the current vs. Vds function (62)

$$I = \frac{\mu_0 C'_{ox} W}{L\left(1+\frac{V_{DS}}{LE_{sat}}\right)} \left(V_{GST} - n\frac{V_{DS}^2}{2}\right)$$

has a nonzero slope for Vds = Vdssat. The slope is

$$\frac{I_{sat}}{LE_{sat} + V_{DSSAT}} \quad (69)$$

The current beyond Vdssat can be approximated by the linear equation

$I = I_{sat} + \left.\frac{\partial I}{\partial V_{DS}}\right|_{V_{DSSAT}} (V_{DS} - V_{DSSAT})$ (70) or

$I = I_{sat}\left(1 + \frac{V_{DS} - V_{DSSAT}}{V_A}\right)$ (71) with the parameter Va (Early voltage)

$V_A = LE_{sat} + V_{DSSAT}$.