Lecture 5

The themes of the lecture are

AC and DC analysis

Inverting Amplifier with feedback

Feedback analysis with Mason's gain formula

AC and DC analysis

The voltages and currents in electronic circuits often contain an average value (the DC value or DC component) that is not zero. This DC component is required to ensure that transistors operate in their correct bias or operating region.

In contrast, the information-carrying portion of a voltage or current typically has a much smaller amplitude; this is called the AC component or small-signal component. The AC component varies with time.

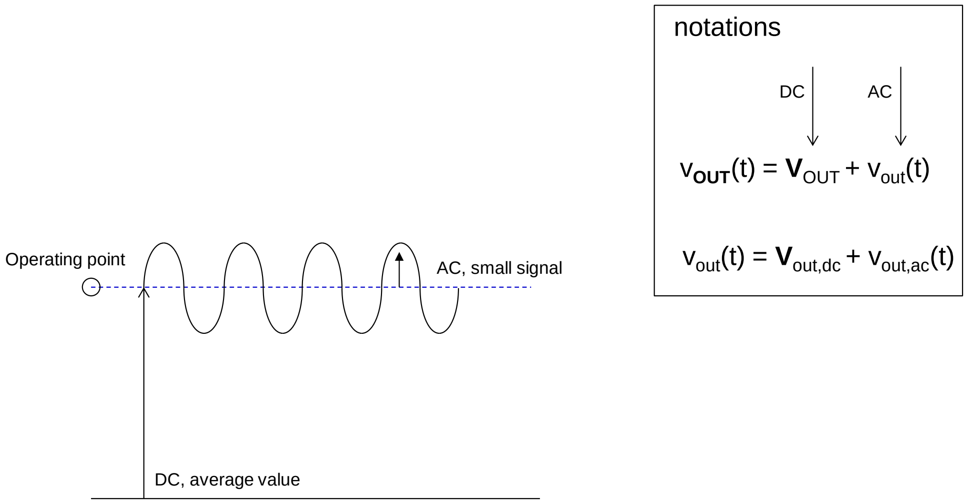


Fig 1: AC and DC

The following notation is used:

$$v_{OUT}(t) = V_{out} + v_{out}(t)$$

$v_{OUT}(t)$ is the actual voltage

V_{out} is the DC part

$v_{out}(t)$ is the small signal (AC part)

If the AC component has a significantly smaller amplitude than the DC component, a small-signal analysis becomes useful. The circuit is first analyzed under static conditions to determine the DC voltages and currents. After that, AC (small-signal) models are derived by linearizing the component characteristics around their operating point. Using these linearized models, the AC voltages and currents can then be calculated.

Note that only the AC component of a voltage or current carries information; the DC component is present solely to ensure the proper operating conditions of the transistors.

An amplifier should ideally not amplify the DC component of a voltage (its average value). It should amplify only the AC component, because this part contains the useful information (signal), Fig 2.

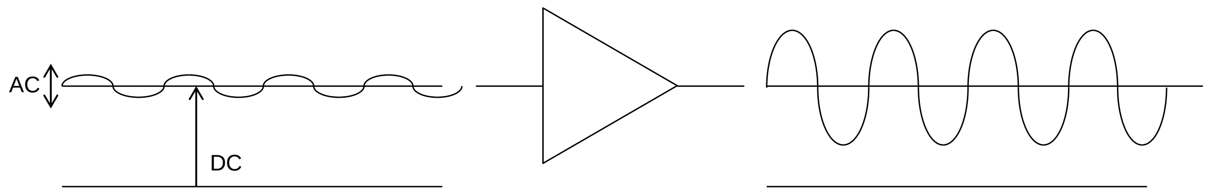


Fig 2: Amplifier amplifies only AC part

An amplifier changes the signal form when its amplification depends on frequency, as shown in Fig 3.

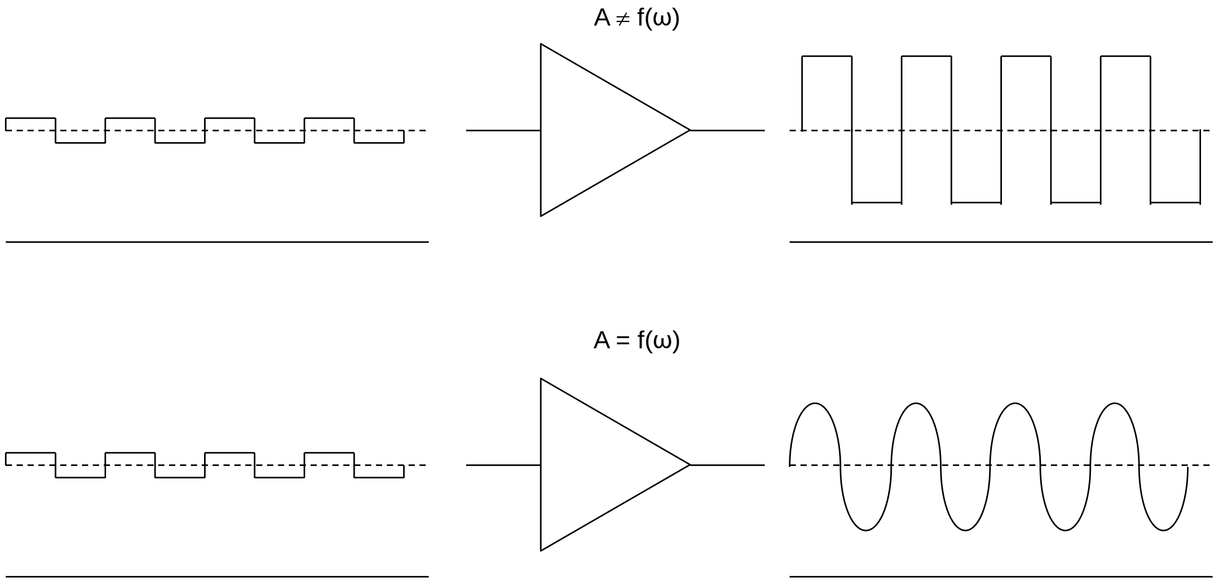


Fig 3: The amplification with and without frequency dependence

Small and large signal models

Electronic circuits are almost always nonlinear; the characteristics are nonlinear and also the capacitances.

To avoid solving full nonlinear differential equations, circuit analysis is typically divided into DC analysis and AC analysis.

$$u_R = Ri_R \qquad i_C = CDu_C \qquad I_C(s) = sCU_C(s) \qquad I_C(i\omega) = i\omega CU_C(i\omega)$$

$$u_L = LDi_L \qquad U_L(s) = sLI_L(s) \qquad U_L(i\omega) = i\omega LI_L(i\omega)$$

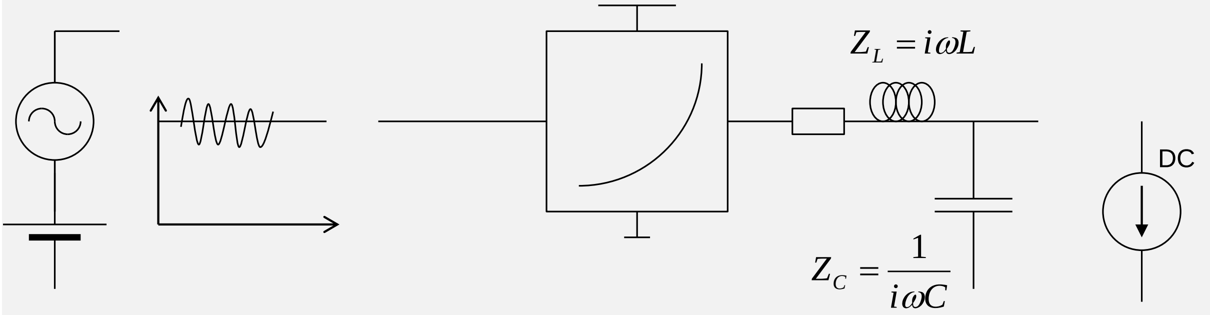


Fig 4: Nonlinear circuits, R, L and C parts equations, impedances

Step 1 DC analysis

We first calculate the DC voltages using the nonlinear component models, with capacitances removed and inductances short-circuited, as shown in Fig 5. (C is removed, L is short circuited)

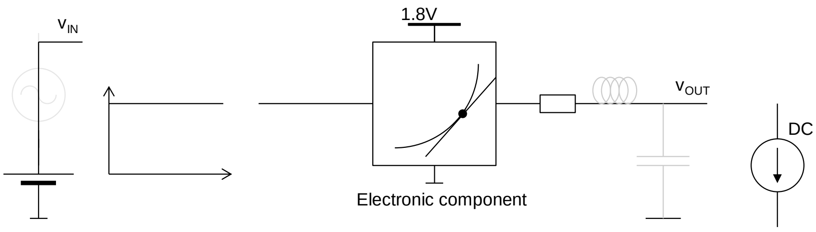


Fig 5: DC analysis, working point

Step 2 AC analysis

Next, the component characteristics and their capacitances/inductances are linearized around the DC operating point. This linearization yields the small-signal models used for AC analysis.

In small-signal analysis, we ignore all DC voltages and currents and calculate only the AC (small-signal) quantities. To obtain the small-signal equivalent circuit, all DC voltage sources are set to zero (replaced by short circuits), and all constant current sources are removed (replaced by open circuits), as showed in Fig 6.

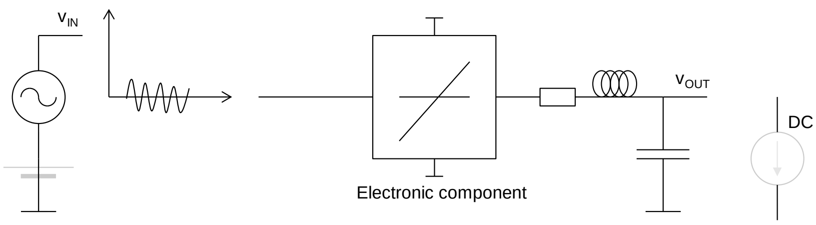


Fig 6: AC analysis

This separation on AC- and DC analysis is not always necessary. Simulation software tools can directly solve nonlinear differential equations. AC and DC analysis is however useful if we want to understand the behaviour of a circuit and for circuit design

Let us start with a basic circuit, the amplifier.

Amplifier

Let us start with a basic circuit: the amplifier. In its simplest form, an amplifier has one input and one output (a single-ended amplifier). The negative supply voltage is connected to ground (GND), and the positive supply voltage is referred to as VDD (e.g. = 1.8V), Fig 7.

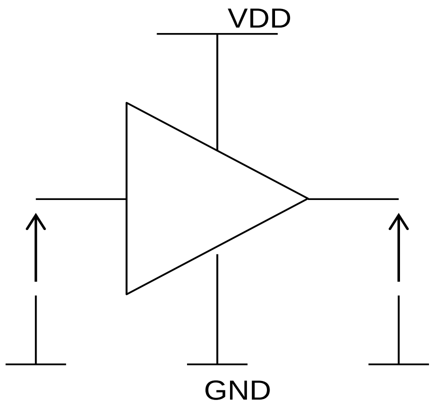


Fig 7: Single ended amplifier

We can characterize an amplifier by applying a voltage source at its input and measuring the output voltage with an ideal voltmeter. Fig 8 shows a typical input–output characteristic of an amplifier. The voltage gain is high (and typically negative) only within a limited portion of this characteristic, which we call the active region. For input voltages that are too high or too low, the amplifier enters saturation, and the gain becomes very small.

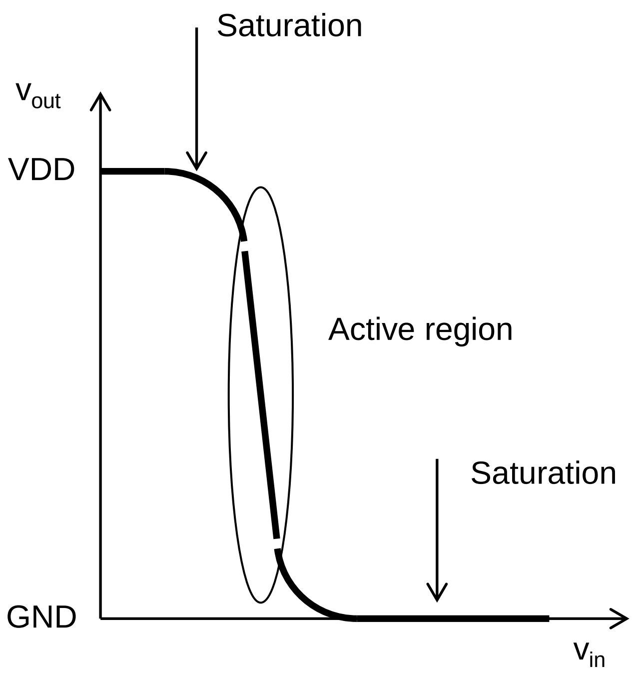


Fig 8: Input output characteristics of the amplifier

NMOS amplifier – DC characteristics

We can implement an amplifier by connecting a load resistor $\mathbf{R_{load}}$ to a transistor $\mathbf{T_{in}}$, as shown in Fig 9.

The resistor has two main tasks:

1) To bias the transistor and establish the correct DC operating point.

2) To convert the output current into a voltage.

This configuration is known as a **common-source amplifier**.

We can determine the input–output characteristic either mathematically, using equations, or graphically, using the graphical device characteristics.

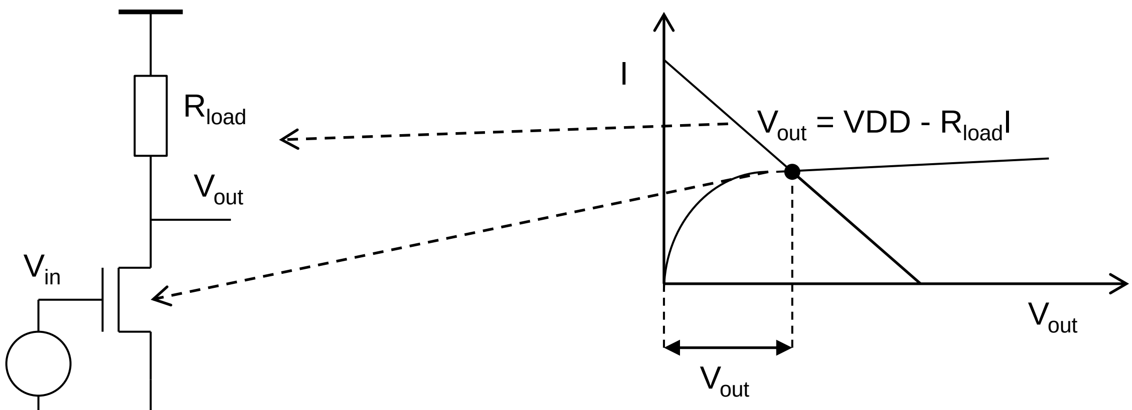


Fig 9: Amplifier realized with transistor and resistor

Graphical analysis

We plot the I_{ds} - V_{ds} characteristics of the transistor and the I-V characteristic of the resistor on the same graph, as shown in Fig 9. The output voltage is found at the **intersection** of the transistor curve and the resistor line. As the input voltage increases, the transistor characteristic shifts upward, causing the output voltage to move from VDD toward GND, as shown in Fig 10. This method of analysis is known as load-line analysis.

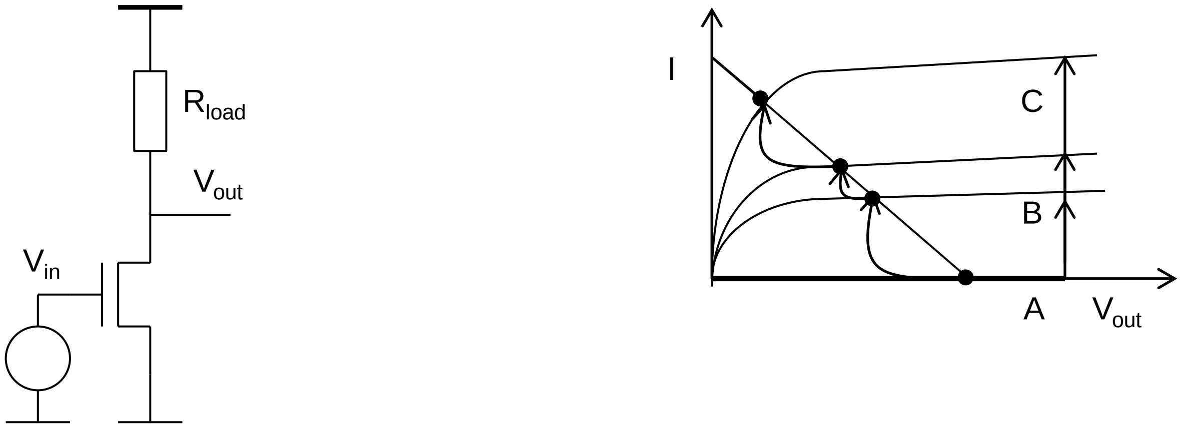


Fig 10: Graphical DC analysis of the amplifier

We distinguish different V_{in} regions of input–output characteristics (Fig 11):

A) The region where the input transistor is off ($V_{gs} = V_{in} < V_{th}$)

B) The region where the transistor is in saturation ($V_{ds} = V_{out} > V_{gs}$-$V_{th} = V_{dssat}$)

C) The region where the transistor is in the triode region ($V_{out} < V_{dssat}$).

Only in region B, the voltage gain is high.

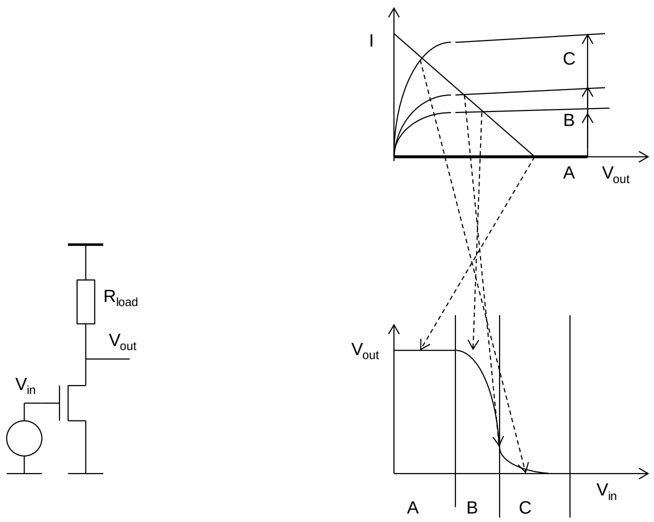


Fig 11: Operating regions of the amplifier

Usage of the amplifier without feedback

We could try the amplifier in an open-loop configuration by connecting a signal source Vs to the input. However, it would not work properly for several reasons. One is illustrated in Fig 12 – the input signal level is too low, leading to saturation of the amplifier.

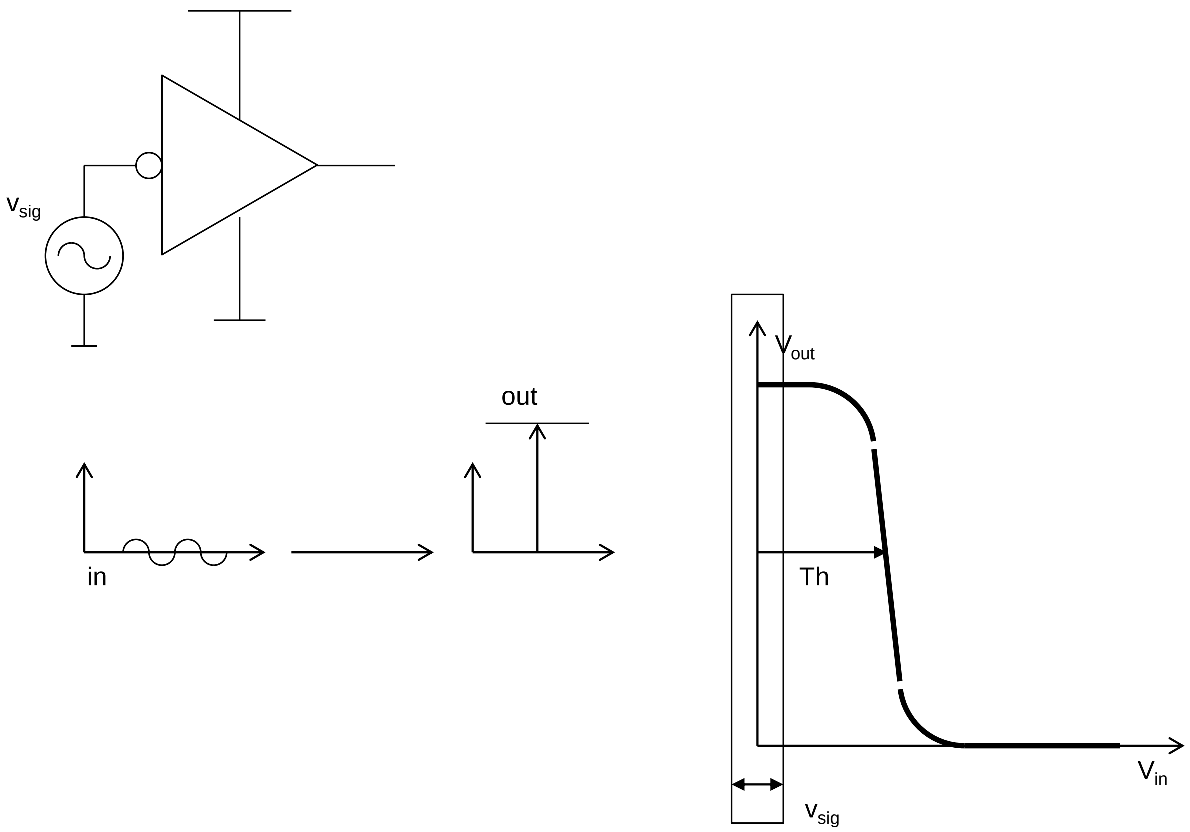


Fig 12: Amplifier without feedback. The input signal is outside of the active region

A signal appears at the output only when the signal source is connected through an additional constant voltage source (VDC). The VDC voltage must be carefully chosen to set the transistor in the active region, (Fig 12).

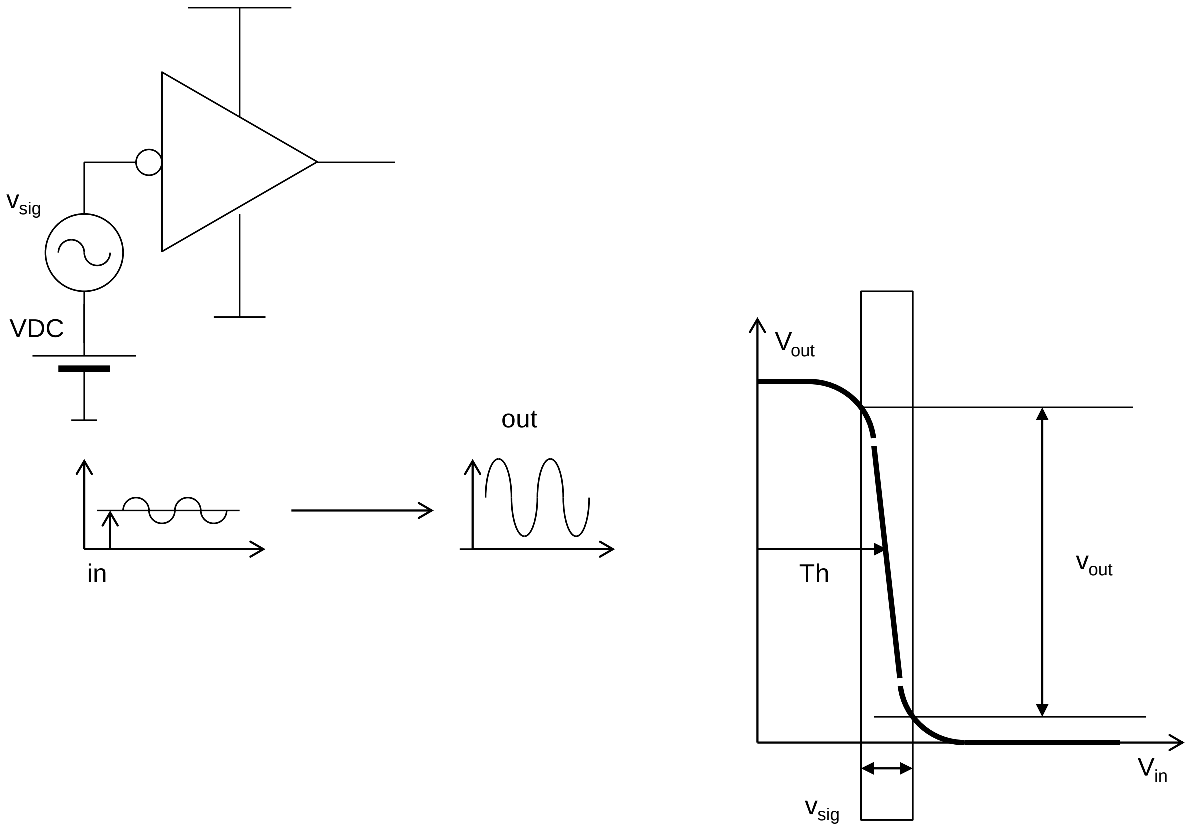


Fig 13: Amplifier without feedback. Bias source VDC assures that the input signal is in the active region

If the input amplitude is too high, the output signals will be distorted, as illustrated in Fig 14.

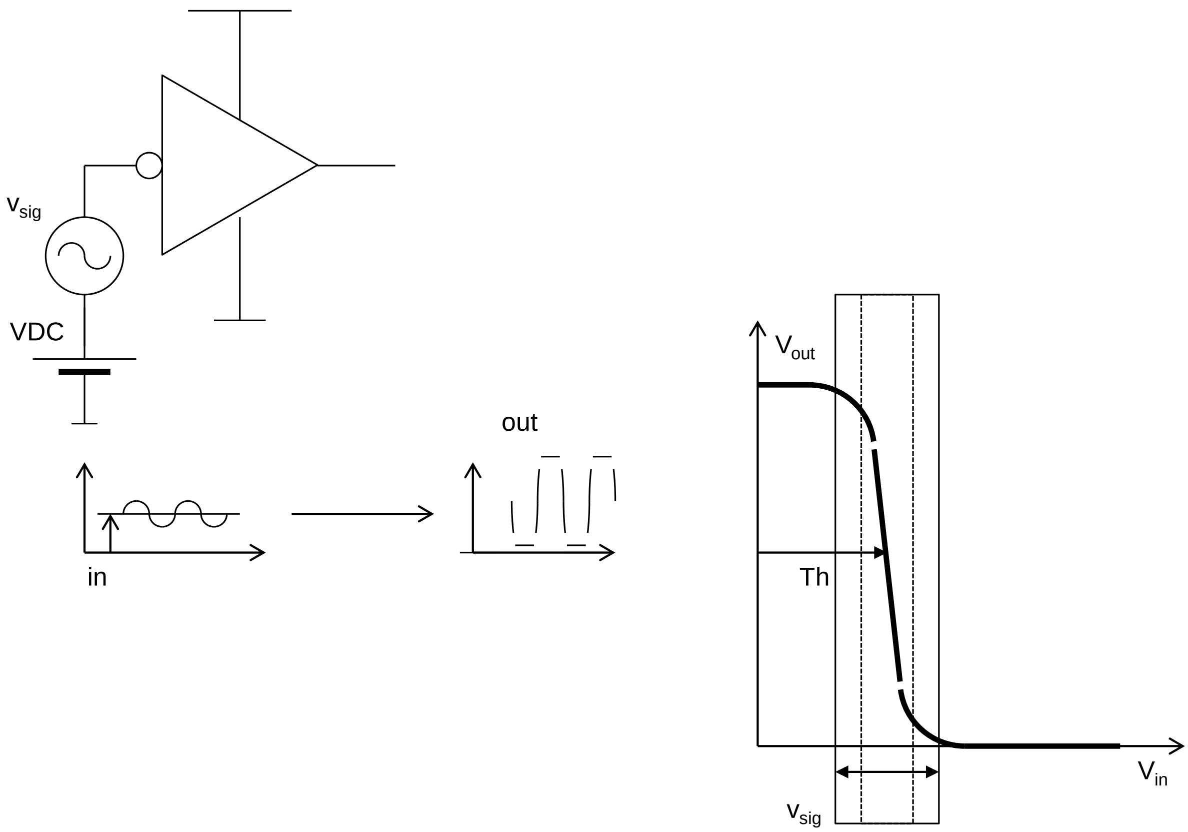


Fig 14: Amplifier without feedback. If the input signal amplitude is large, output signal is distorted

Changes in temperature affect the amplifier's behaviour. Temperature variations can cause the input signal to leave the active region, resulting in a weaker output signal, Fig 15.

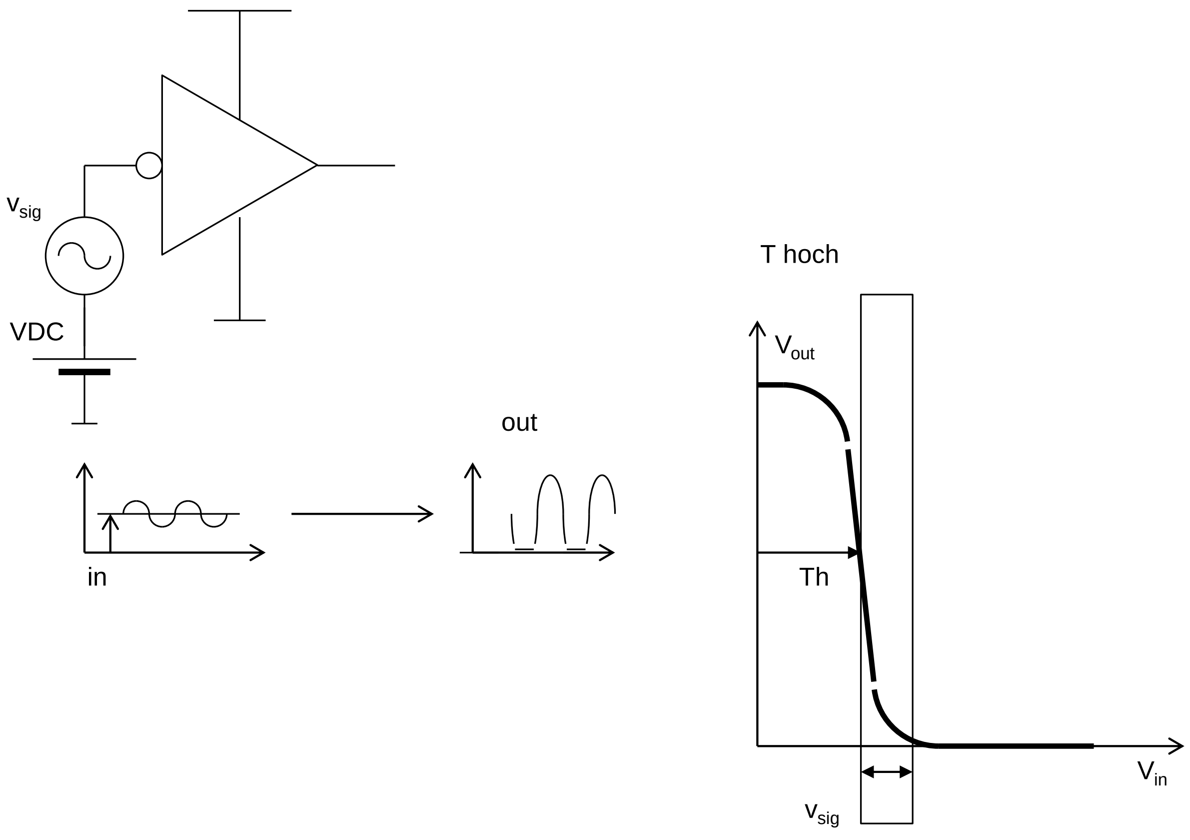


Fig 15: Amplifier without feedback. Temperature influences the amplifier characteristics

Amplifier with feedback

Feedback is an important technique that converts nonlinear active components into well-behaved linear amplifiers. It enables the design of precise amplifiers and oscillators.

In this lecture, we will present one of the basic circuits that uses feedback — the **inverting amplifier**.
We will examine two versions of an inverting amplifier with feedback.

First variant: amplifier with continuous feedback

In Fig 16, the circuit employs a slow and "strong" resistive feedback path for DC voltages and a faster, "weaker" capacitive feedback path for AC signals. In this configuration, slow-changing signals are not amplified; the DC feedback is used solely to stabilize the operating point. The AC signals, however, are amplified.

The circuit consists of a feedback resistor R_{fb} and two capacitors C_{in} and C_{fb}. The AC input signal is applied through C_{in}. DC and AC analyses are performed separately.

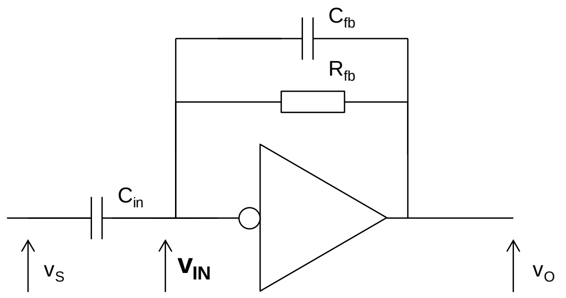


Fig 16: Inverting amplifier with continuous feedback

The second variant of the inverting amplifier is shown in Fig 17. It is known as the switched amplifier (or switched-capacitor amplifier). In this design, a switch is used instead of a resistor. The switching action is employed to establish and maintain the correct operating point.

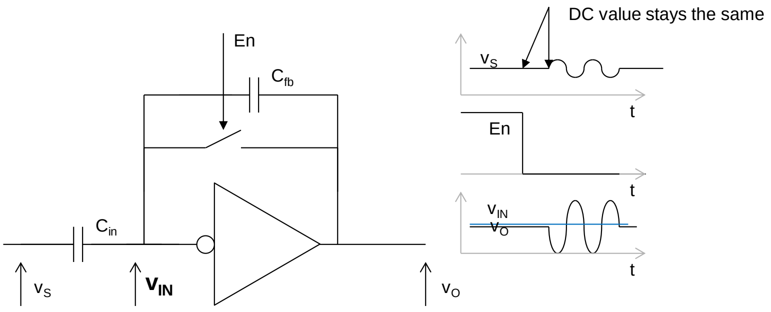


Fig 17: Switched inverting amplifier

DC analysis

Let us now do the DC analysis of the feedback-amplifier. The **working point** of the amplifier must be determined.

Step 1: Remove the capacitors. The DC circuit then consists of the amplifier and the feedback network, as shown in Fig 18.

Since the amplifier has **infinite input impedance**, no DC current flows through R_{fb}. Therefore, the DC condition becomes:

$V_{out} = V_{in}$.

The DC analysis is identical for both amplifier variants (continuous-feedback and switched).

In the continuous-feedback amplifier, R_{fb} is an ordinary resistor.
In the switched amplifier, R_{fb} corresponds to the **on-resistance** of the switch.

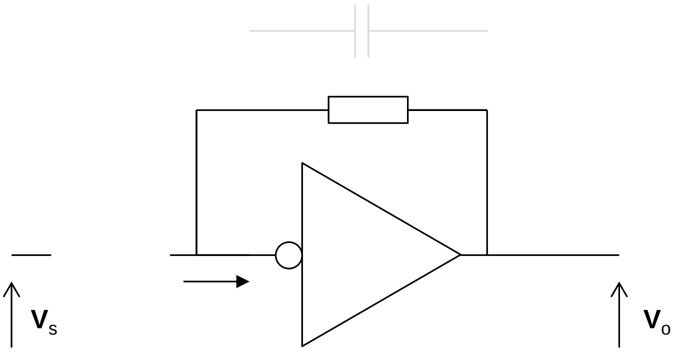


Fig 18: Inverting amplifier DC analysis

We plot the **characteristics of the amplifier** and the **feedback network** on the same graph. The **intersection point** of the two curves represents the **operating point** of the amplifier, Fig 19.

It is important to determine whether the **feedback is positive or negative**.

Rule of thumb: If there is **only one intersection point (**Fig 19**)**, the feedback is **negative**, and the operating point is **stable**. For a small disturbance (e.g., noise voltage **v_n**), the **negative feedback acts to counteract** the disturbance.

Positive feedback often results in multiple intersection points, corresponding to several possible stable operating points, Fig 20. The intermediate operating point (B) is unstable. A small disturbance (e.g., noise voltage vn) is amplified, pushing the circuit toward one of the stable operating points.

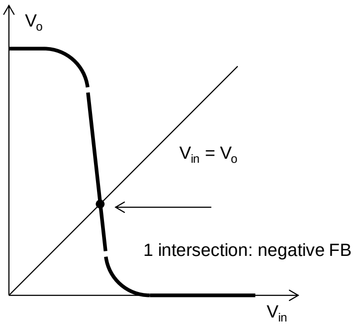


Fig 19: Inverting amplifier DC analysis, graphical solution

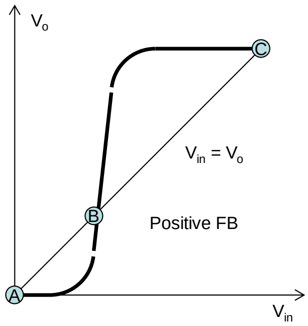


Fig 20: Positive feedback

Fig 21 shows that the working point of our amplifier is set in the active region of the characteristics, where the amplifier exhibits high gain.

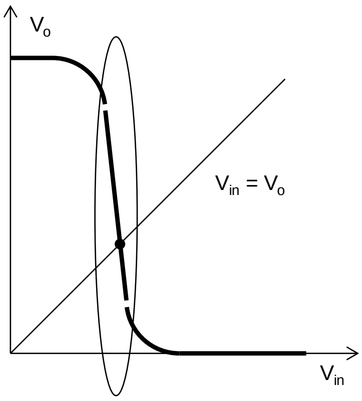


Fig 21: Operating point

Derivation of the small-signal model of the amplifier

We can obtain the small-signal model of the amplifier in two ways:

1. by linearizing its input–output characteristic, or
2. by directly using the small-signal model of the transistor.

In this lecture, we derive the small-signal model of the transistor (for slow signals) starting from the transistor equation:

$$I_{ds} = I_{dssat}(Vgs)(1 + (V_{ds} - V_{dssat})/V_A)$$

We linearize the equation in the working point:

$$i_{DS} = I_{ds,DC} + i_{ds,ac} = I_{ds,DC} + \frac{dI_{ds}}{dV_{gs}} v_{gs} + \frac{dI_{ds}}{dV_{ds}} v_{ds} + \frac{dI_{ds}}{dV_{bs}} v_{bs}$$

It follows:

$$i_{ds,ac} = \frac{dI_{ds}}{dV_{gs}} v_{gs} + \frac{dI_{ds}}{dV_{ds}} v_{ds} + \frac{dI_{ds}}{dV_{sb}} v_{sb} = g_m v_{gs} + g_{ds} v_{ds} + g_{mb} v_{bs}$$

The last term describes the substrate effect and can be neglected.

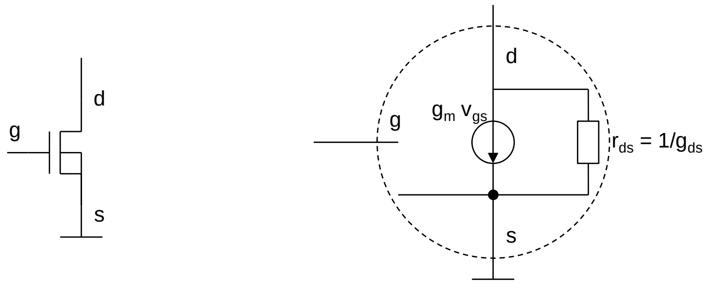


Figure 22: Small signal model of the transistor

Replacing the transistor with its small-signal model gives the small-signal model of the amplifier. All constant voltage sources, including VDD, are switched off.

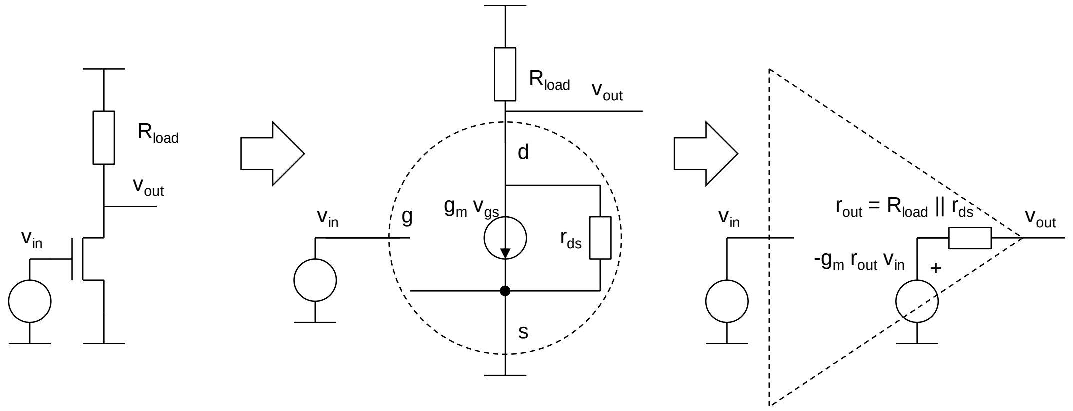


Figure 23: Small-signal model of the amplifier

The following figures illustrate the AC analysis of the inverting amplifier from Fig 16. We assume that R_{fb} is very large. Under this condition, the impedance of capacitor C_{fb}, given by $1/(\omega C_{fb})$, is much smaller than R_{fb} for all frequencies of interest. Since C_{fb} and R_{fb} form a parallel network, R_{fb} can be neglected.

The resulting simplified small-signal circuit is shown in Fig 24.

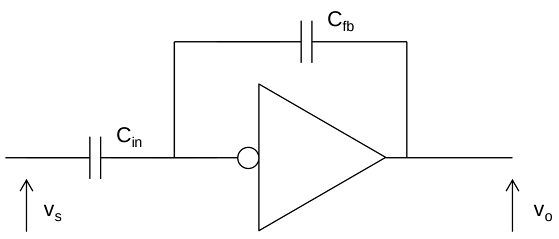


Fig 24: Small signal analysis of the inverting amplifier

For the switched amplifier, we assume that signals are generated after the switch is opened. Therefore, R_{fb} can by also neglected in the AC analysis, Fig 17.

The following analysis is valid for both cases, the continuous and the switched amplifier.

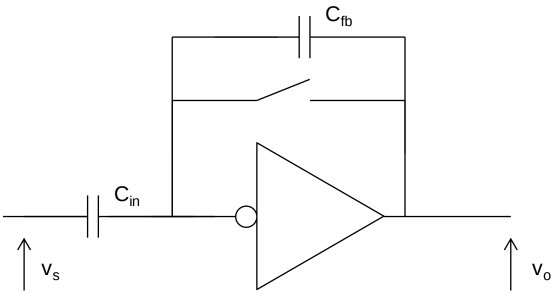


Fig 25: Switched amplifier

The feedback is negative, as illustrated in Fig 26.

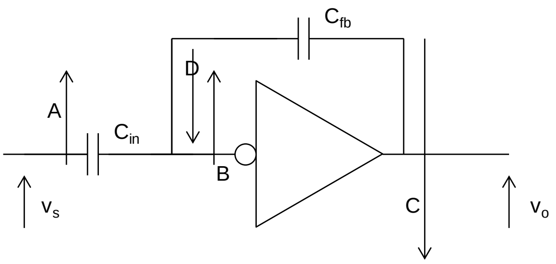


Fig 26: If we increase voltage A, B is increased too. C decreases, as well as D. Therefore D acts against input signal. It is negative feedback.

The feedback circuit in our amplifier operates as follows: the voltage at the amplifier output is converted into a **feedback current** by the capacitor C_{fb}. This feedback current is then subtracted from the input signal current. This type of feedback is known as **voltage-to-current feedback**. The passive network at the amplifier input, which combines the input signal and the feedback signal, is called an **adder**, as shown in Fig 27.

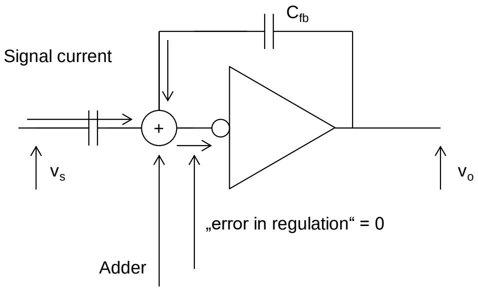


Fig 27: Illustration of voltage-current feedback

Let us calculate the amplification: v_{out}/v_{sig}.

First approach: We could calculate the currents and voltages using, for example, the **node-voltage method**, as illustrated in Fig 28.

Writing the equations is relatively straightforward. In our circuit, there are only two independent node potentials, v_{in} and v_{out}, as shown in Fig 28. There is a third node (node “X”), but its voltage depends on v_{in}. However, solving these equations can be tedious. The resulting expressions are long and can only be simplified at the very end. Including R_{fb} in the calculation would make the process even more cumbersome.

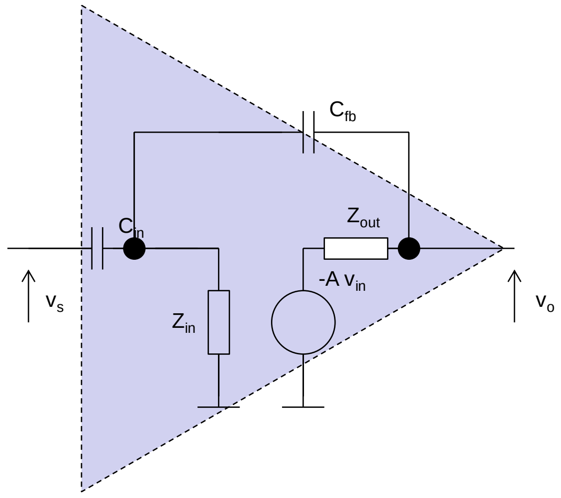


Fig 28: Method of node potentials

For these reasons, we use an alternative method to analyze feedback circuits, based on the following formula:

Mason's Formula - Derivation

The AC circuit of the voltage amplifier is a **linear circuit** consisting of the input v_S, the output v_O the amplifier, and the feedback network (Fig 30).

Let us derive $v_O = f(v_S)$.

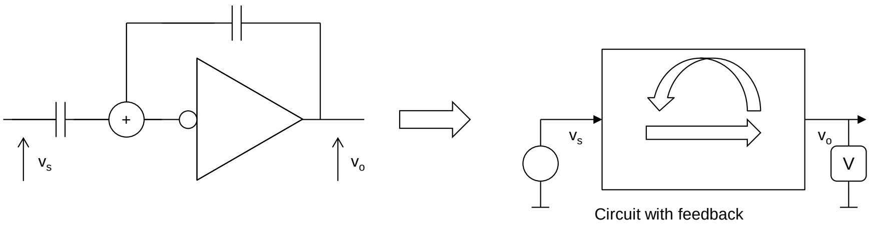


Fig 29: Small Signal Model of the Inverting Voltage Amplifier

To simplify the analysis, we use the following technique.

We “cut” the line at the input of the amplifier, as shown in Fig 31. It is important that the current through the line being cut was **zero** before disconnecting it.

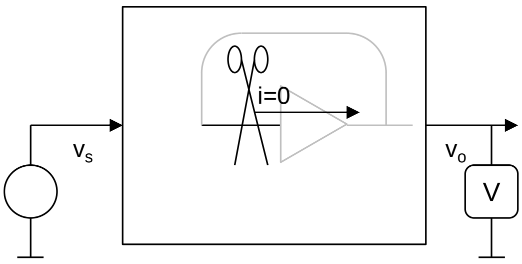


Fig 30: Feedback is disconnected

An additional voltage source, v_{in}* is introduced, and a corresponding additional output, v_i is defined (Fig 32).

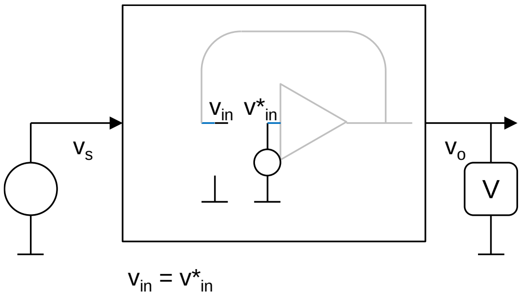
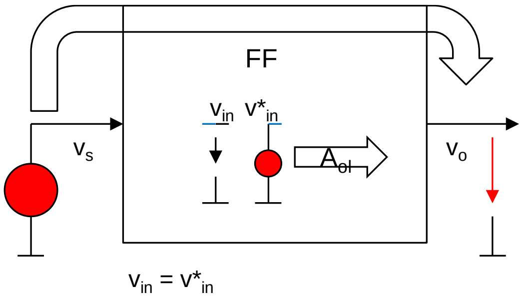


Fig 31: Additional source at the output

Let us calculate v_o.

The test circuit for calculating v_o is shown in the next figure:



$$v_o = FFv_s + A_{ol}v_{in}*$$

Test circuit for v_0

The following equation can be written:

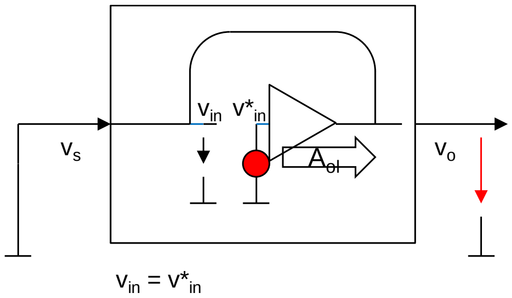
$v_o = FFv_s + A_{ol}v_{in}*$ (A)

The coefficients in the circuit are defined as follows:

FF – feed forward

A_{OL} – open loop gain

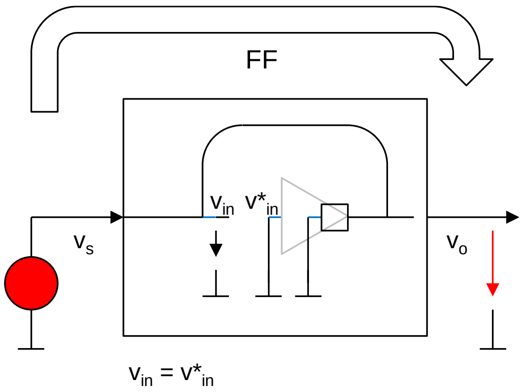
The next two figures show the test circuits used to calculate A_{ol} and FF.



$$v_o = FFv_s + A_{ol}v_{in}^*$$

0

Test circuit for calculation of A_{ol}

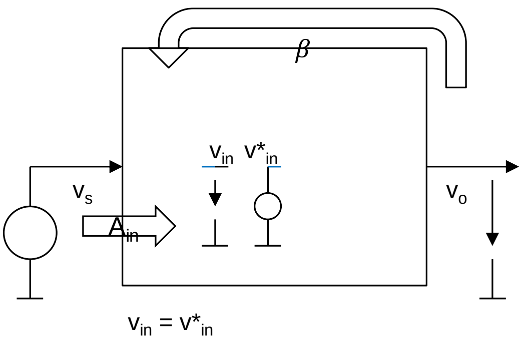


$$v_o = FFv_s + A_{ol}v_{in}^*$$

0

Test circuit for calculation of FF

Now let us take a look at the second output v_{in}.



$$v_{in} = A_{in}v_s + \beta v_o$$

Test circuit for v_{in}

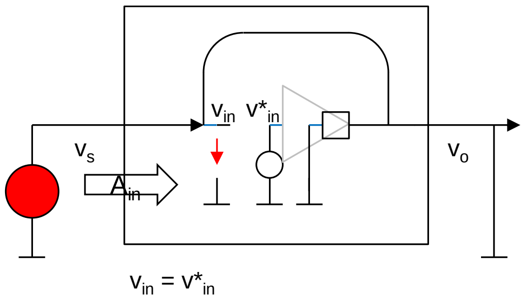
The following equation can be written:

$v_{in} = A_{in} v_s + \beta v_o$ (B)

A_{in} – input gain

β – feedback

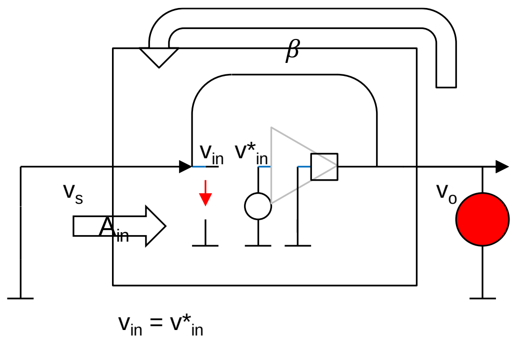
The next two figures show the test circuits used to calculate A_{in} and β.



$$v_{in} = A_{in} v_s + \beta v_o$$

0

Test circuit for calculation of A_{in}



$$v_{in} = A_{in} v_s + \beta v_o$$

0

Test circuit for calculation of β

If we consider the condition $v_i = v^*_i$ we can derive the **transfer function** of the original circuit from the equations A and B:

$$v_o = v_s \frac{FF + A_{in}A_{ol}}{1 - \beta A_{ol}} \qquad (1)$$

We also define βA – loop gain – as:

$$\beta A \equiv \beta A_{ol}$$

In the case of negative feedback, βA is negative (more precisely: DC part of βA is negative).

Equation (1) is known as Mason's formula. It is particularly useful because it is usually easier to calculate the coefficients FF, A_{ol}, A_{in}, β and then insert them into the formula than to solve the original circuit directly.

Mason's Formula – Summary

$$v_o = v_s \frac{FF + A_{in}A_{ol}}{1 - \beta A_{ol}};\ \beta A \equiv \beta A_{ol} \qquad (2)$$

FF – feed forward

A_{ol} – open loop gain

A_{in} – input gain

β – feedback

βA – loop gain

For the calculation of factors A_{in}, A_{ol}, FF and βA, the **feedback loop** should be cut at a suitable point, usually at the **input of the amplifier**, as shown in Fig 32.

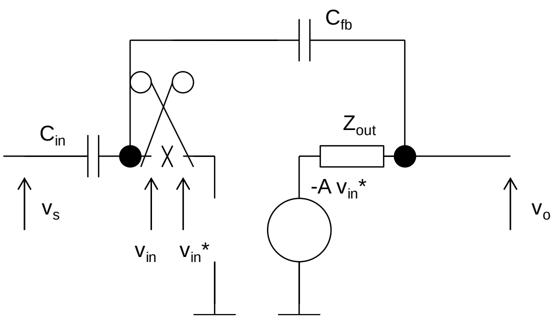


Fig 32: The feedback is cut at the input of the amplifier

A_{in} is the amplification in the "feedback-adder". It is measured from the signal source (the main input) to the input of the amplifier, as Fig 33 shows.

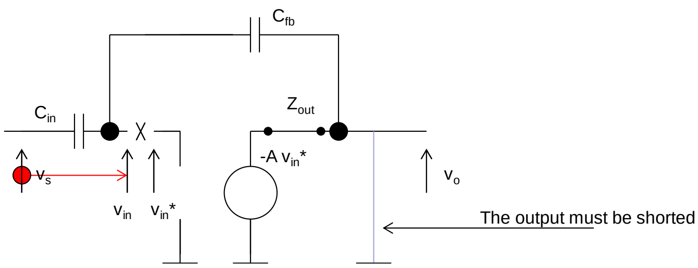


Fig 33: Input gain

A_{ol} is the open loop gain. It is measured from the amplifier input to the main output, as Fig 34 shows.

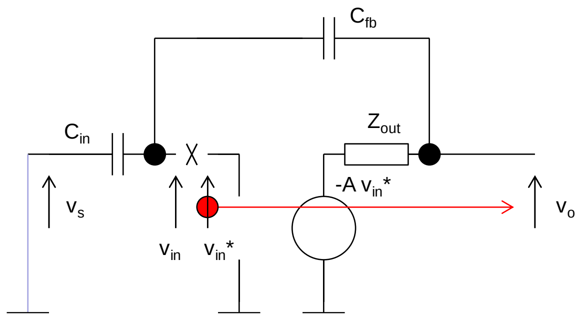


Fig 34: Open loop gain

FF (Feedforward) describes the signal transmission from the main input to the main output through the feedback network when the amplifier is switched off. FF is usually negligible.

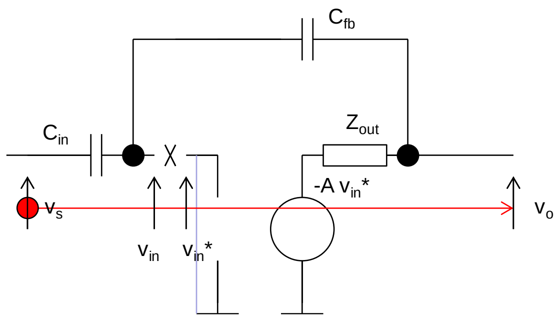


Fig 35: Feed forward

BA is the loop gain. It is the amplification from the input of the amplifier to the point before the intersection point, as shown in Fig 36. In case of a negative feedback βA is negative (more precisely: DC part of βA is negative).

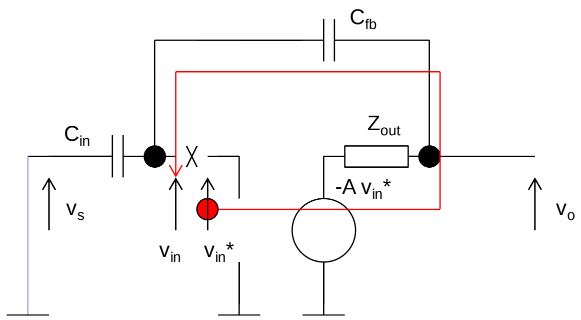


Fig 36: Loop gain

It holds $\beta A = \beta \times A_{ol}$.

β is the feedback from the main output to the amplifier input, Fig 37.

Now, we calculate the individual terms. We use four simplified schematics to facilitate the calculations.

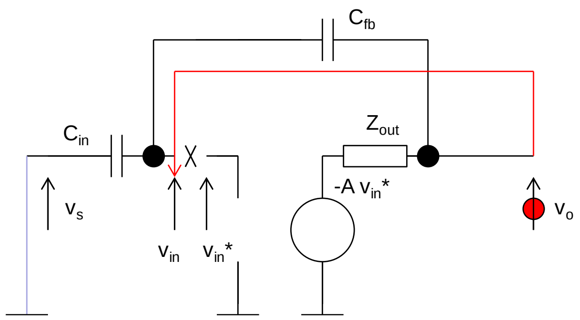


Fig 37: Feedback

Let us first calculate A_{in} (Fig 38).

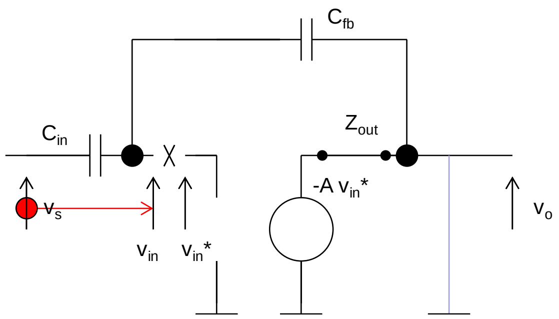


Fig 38: Calculation of A_{in}

A_{in} is defined as v_{in}/v_{sig}.

It is important to note that the **main output must be short-circuited** when calculating A_{in}. (In our case, the output is a voltage; if the output were a current, the output line would need to be opened instead.) With this simplification, the circuit reduces to a simple **voltage divider**, making the calculation straightforward (Fig 38).

$$v_{in} = \frac{Z_{FB}}{Z_{FB} + Z_{in}} v_s$$

$$A_{in} = \frac{Z_{FB}}{Z_{FB} + Z_{in}}$$

Let us calculate A_{ol} (Fig 39). It is defined as a v_{out} / v_{in}*. Between v_{in}* and v_{out} , the circuit consists of the amplifier with a **voltage divider** at the output (Fig 39).

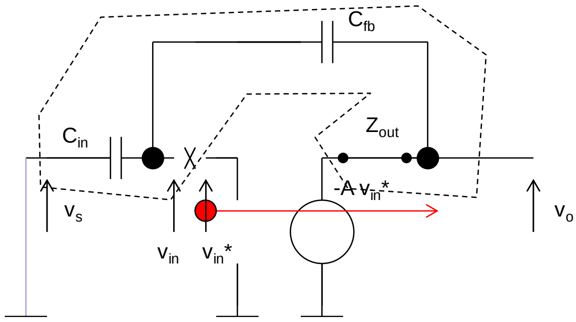


Fig 39: Calculation of A_{ol}

When we assume $R_{out} = 0$, we obtain $A_{ol} = -A$. Otherwise it would be

$A_{ol} = -A (z_{in} + z_{fb}) / (z_{in} + z_{fb} + z_{out})$.

The advantage of this method is that the factors can be **simplified from the very beginning**. In particular, the **voltage-divider formula** is frequently used to make the calculations easier.

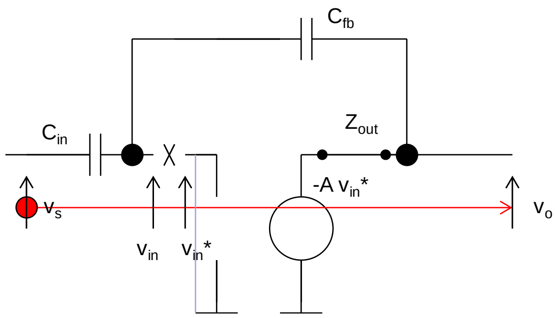


Fig 40: Calculation of FF

Let us calculate **FF**. For this calculation, the amplifier must be **switched off**, which is achieved by shorting the input voltage v_{in}*. The circuit gets simpler as shown in Fig 41. FF is defined as: v_{out}/v_{signal}. We calculate FF = $z_{out}/(z_{in} + z_{fb} + z_{out})$. For $R_{out} \sim 0$, FF ~ 0.

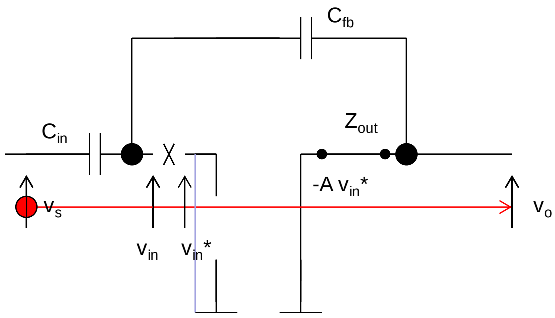


Fig 41: Calculation of FF (2)

Finally, the loop gain is calculated, Fig 42. BA is defined as v_{in} / v_{in}*.

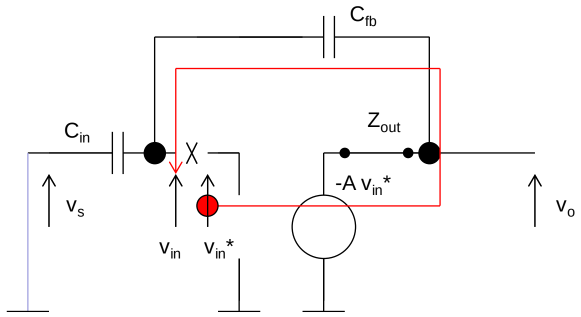


Fig 42: Calculation of loop gain

The result is:

$$\beta A = -A \frac{Z_{in}}{Z_{in} + Z_{FB} + Z_{out}}$$

for $Z_{out} = 0$ it is:

$$\beta A = -A \frac{Z_{in}}{Z_{in} + Z_{FB}}$$

Finally, by substituting the calculated values into Mason's formula (1), we obtain the amplifier gain with feedback:

$$A_{FB} = \frac{-A \frac{Z_{FB}}{Z_{FB} + Z_{in}}}{1 + A \frac{Z_{in}}{Z_{FB} + Z_{in}}}$$

or

$$A_{FB} = \frac{-A \frac{C_{in}}{C_{FB} + C_{in}}}{1 + A \frac{C_{FB}}{C_{FB} + C_{in}}}$$

A common assumption is $|\beta A| >> 1$. This is valid when the **active amplifier has a large gain**. Under this condition, the gain with feedback **simplifies significantly**.

$$A_{FB} = -\frac{A_{in} A_{ol}}{\beta A_{ol}} = -\frac{A_{in}}{\beta}$$

or

$$A_{FB} = -\frac{C_{in}}{C_{fb}}$$

It is both interesting and practical that the **amplification with feedback** A_{fb} does **not** depend on the open-loop gain βA. The open-loop gain is difficult to control and is temperature dependent. In contrast, the **passive components**, especially capacitors, have stable and well-known properties. Since A_{fb} depends only on the capacitances, high accuracy and stability can be expected.

Accuracy can be further improved by implementing the two capacitors in the same way (for example, as metal–metal capacitors), which ensures matching and reduces errors.

The gain with feedback is reduced by βA compared to the value without feedback. In this sense, the amplifier is "sacrificed" to stabilize the circuit.

Virtual ground

The voltage at the input of the amplifier is practically 0, which follows from the formula

$v_{in} = v_{out} / A = - v_{sig}\ C_{in}/(C_{fb}\ A)$.

In other words, there is no (AC) voltage at the amplifier input. The input is a virtual ground. In well-built systems with feedback, the error term is ~ 0.

Notice that we could have assumed $v_{in} = 0$ at the beginning, in order to simplify the circuit analysis. It is easy to obtain $v_{out} = - Z_{fb}/Z_{in}\ v_{sig}$ directly if $v_{in} = 0$ is assumed. The voltages form a kind of lever as shown in Fig 43.

However, in the case of a simplified analysis (assuming $v_{in} = 0$), we would not know which condition A should meet, or how large it must be.

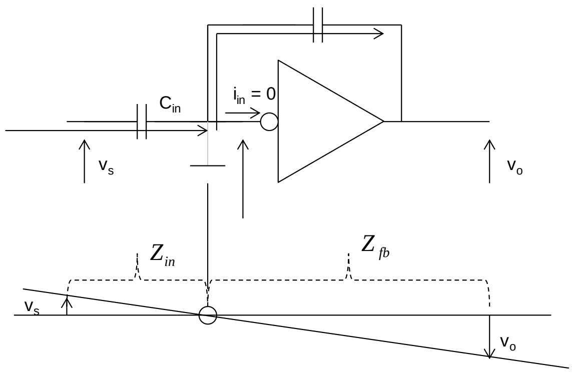


Fig 43: Virtual ground

Importance of open loop gain

Let us assume the condition βA >> 1, d.h. $A\ C_{fb} / (C_{in} + C_{fb}) >> 1$.

From the formula $A_{FB} = A_{IN}/\beta$ and $\beta A = \beta A_{OL} >> 1$ following equation can be derived:

$\beta A = \beta A_{OL} = (A_{IN}/A_{FB})\ A_{OL} >> 1$

or

$A_{OL} >> A_{FB} / A_{IN}$

If we want a specific gain with feedback (e.g. $A_{FB} = 10$), the open loop gain must be at least 1-2 orders of magnitude higher than A_{FB} / A_{IN}.

Feedback - Summary

This paragraph briefly explains the idea of using feedback in the amplifier.

This paragraph briefly explains the concept of using feedback in the amplifier.

We implement automatic regulation of the input voltage v_{IN} through negative feedback (Figure 44).

The voltage v_{IN} is controlled so that the amplifier operates in the active (linear) region of its characteristics, ensuring a high gain.

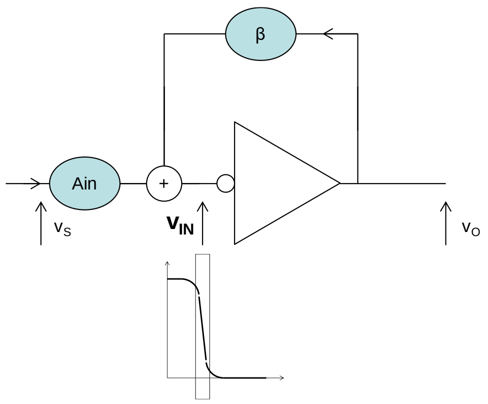


Figure 44: Feedback

Feedback enables signal amplification in the following way:

It holds (Figure 44):

$$v_{IN} = A_{IN} v_S + \beta v_O$$

The feedback regulates v_{IN}. For this reason, v_{IN} is constant. It follows:

$$dv_O = \frac{-A_{IN}}{\beta} dv_S$$

The small signal gain is:

$$A_{FB} = \frac{dv_O}{dv_S} = \frac{-A_{IN}}{\beta}$$

The parameters A_{IN} und β are often frequency dependent.

For low frequencies, parameter β is large and A_{IN} small.

Since β und A_{IN} are realized with passive components, we have in this case:

$\beta_{max} = 1$ and $A_{IN,min} = 0$

For high frequencies β is small (<< 1) and A_{IN} large (~ 1).

The small signal gain is:

$$|A_{FB}| = \frac{A_{IN}}{\beta} \gg 1$$