Lecture 3

The following topics will be covered in this lecture

Drain-current as a function of drain-source voltage

Saturation of the current

Summary of MOSFET equations

Symmetrical Formula for triode- and saturation region

Drain-source current

We consider an NMOS transistor. Let us calculate the transistor current for small voltages V_{ds}

Assume that we have a channel of electrons between the source and the drain (Fig 1). Channel is an ohmic connection (a resistor) between source and drain. We know the total charge within the channel:

$Q = C_{ox}(V_{gs} - V_{thsb})$ (1)

The channel has dimensions: length L, width W, thickness t_{chan}.

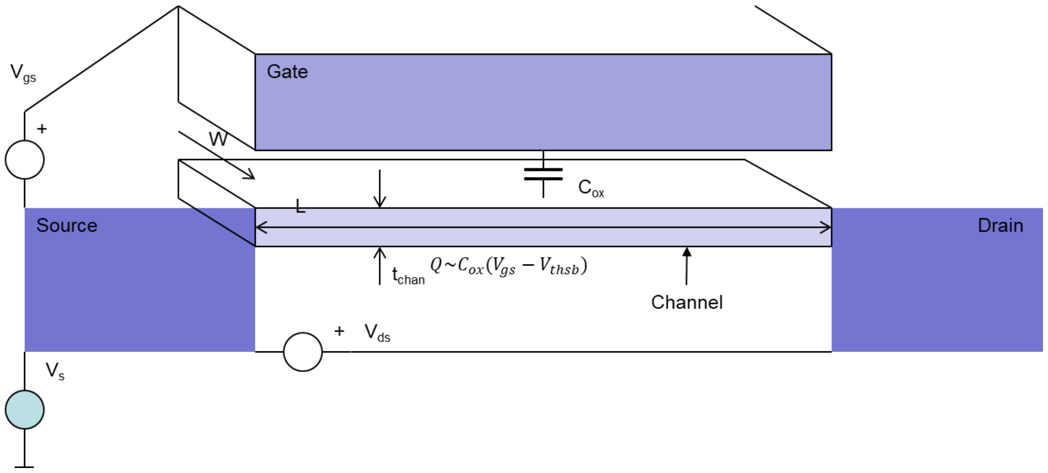


Fig 1: Channel is an ohmic connection between source and drain

If we have a voltage between drain and source (V_{ds}), a current flows from drain to source (I_{ds}). The conventional (technical) current direction is from drain to source, in the case of NMOS transistor, electrons in the channel are moving from source to drain.

The channel forms a resistance, the current is given by the following equation (Fig 1):

$$I_{ds} = A\, e\, \mu\, n\, E$$

e is the elementary charge, μ is mobility, n is electron density in the channel, E is the horizontal E-field component, A is the channel cross section, which is equal to channel width W multiplied by channel thickness t.

It follows:

$$I_{ds} = W\, t\, e\, \mu\, n\, E$$

It can be derived:

$$n\,t\,e = \frac{Q}{WL} = \frac{C_{ox}\,(V_{gs} - V_{thsb})}{WL}$$

L is the length of the channel. (We use the result Q = Cox (V_{gs} - V_{thsb}))

We get:

$$I_{ds} = \mu\, C'_{ox}\, W\left(V_{gs} - V_{thsb}\right) E$$

C'$_{ox}$ is the capacitance per unit area.

E-field is nearly:

E = V_{ds}/L.

Therefore:

$$I_{ds} = \mu C'_{ox}\frac{W}{L}\left(V_{gs} - V_{thsb}\right)V_{ds} \qquad (2).$$

This is the simplest equation for the transistor current.

It is valid for small V_{ds}. The assumption was that the charge is evenly distributed in the channel. The charge density and channel thickness are constant across x.

We see that the current depends on the W/L ratio. This is typical for MOSFETs. In contrast to that, in the case of bipolar transistors, the size does not influence current.

Saturation

How does the current increase when the V_{ds} become larger?

The charge in the channel is expressed with the formula (1)

$Q = C_{ox}(V_{gs} - V_{thsb})$

This formula holds when V_{ds} is zero or small.

What happens if we have V_{ds} >> 0?

We can expect following: The channel charge near source is Cox (V_{gs} -V_{thsb}) and the channel charge near drain is C_{ox} (V_{gd} -V_{thdb}). This reflects the symmetry of the structure.

It holds (see Lecture 2):

$$V_{thdb} = V_{th} + (n-1)V_{db} = V_{thsb} + (n-1)V_{ds}$$

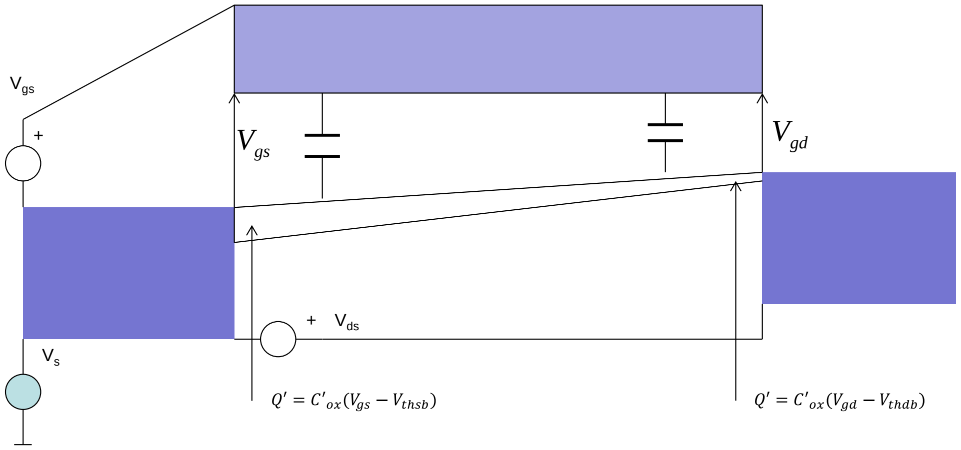


Fig 2: Current as function of V_{ds}. $V_{ds} > 0$

Therefore for a drain voltage $V_{gd} = V_{thdb}$ we have virtually no channel on the drain side. We say that the channel is pinched-off. The current is still flowing because the E-field is high close to the drain.

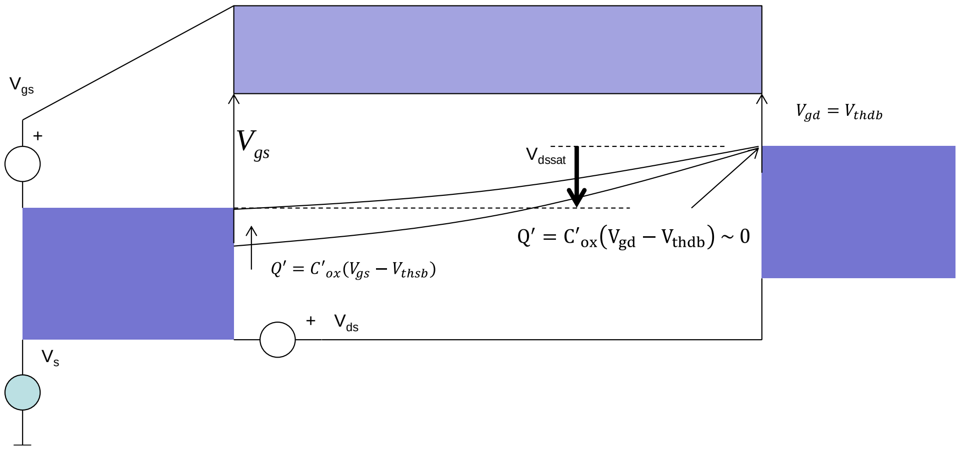


Fig 3: Current as function of V_{ds}. $V_{ds} = V_{dssat}$

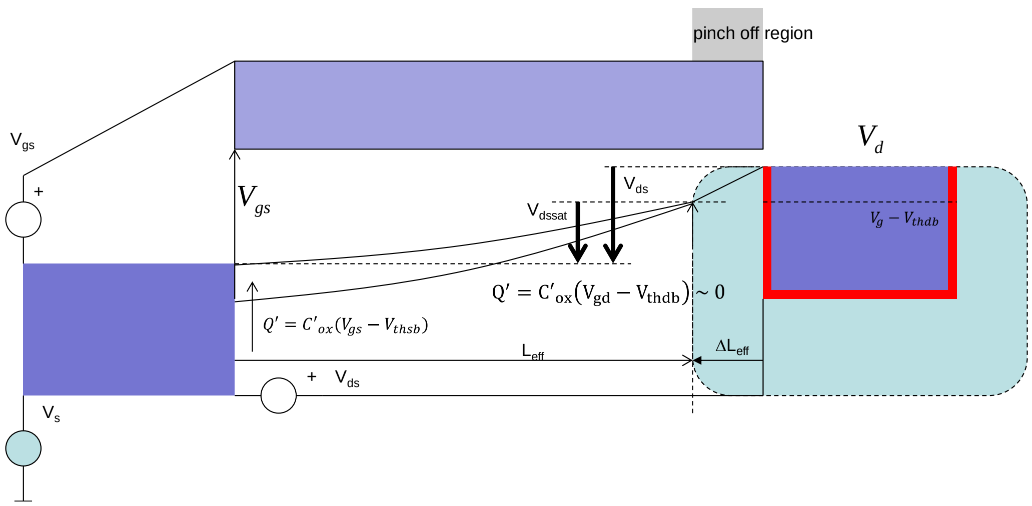


Fig 4: Current as function of V_{ds}. $V_{ds} > V_{dssat}$

The condition for saturation: $V_d = V_g - V_{thdb}$ oder $V_{gd} = V_{thdb}$ can be expressed as:

$V_{gs} - V_{ds} = V_{thdb}$ (both sides - V_s) or as $V_{ds} = V_{gs} - V_{thdb} = V_{gs} - V_{thds} - (n-1)V_{ds}$.

Therefore $V_{ds} = (V_{gs} - V_{thsb})/n$

We define saturation voltage as

$$V_{dssat} = (V_{gs} - V_{thsb})/n \qquad (3)$$

For higher V_{ds} the current increases because the pinch-off zone gets larger. The effective channel length gets smaller, which causes current increase. The pinch-off region grows because the voltage $V_{ds} - V_{dssat}$ must be dropped across it.

What is the value of the drain source current for $V_{ds} = V_{dssat}$? (at the beginning of saturation)

Let us make a not so correct assumption (A1): the formula (2), which we have derived for small V_{ds}, applies from $V_{ds} = 0$ to $V_{ds} = V_{dssat}$.

The drain source current for V_{dssat} voltage (saturation current) can be calculated from the equation (2) by using $V_{ds} = V_{gs} - V_{thsb}/n$.

$$I_{dssat} = \mu C'_{ox} \frac{W}{nL} \left(V_{gs} - V_{thsb}\right)^2 \qquad (4)$$

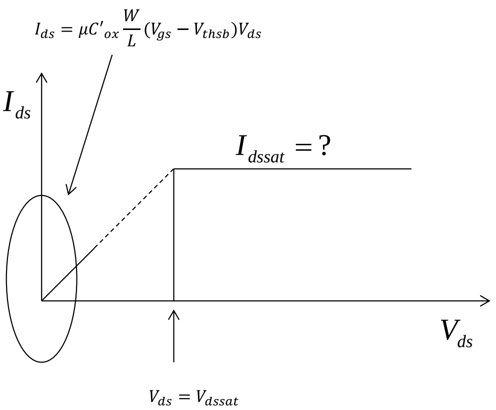


Fig 5: I_{ds} formula – simple derivation

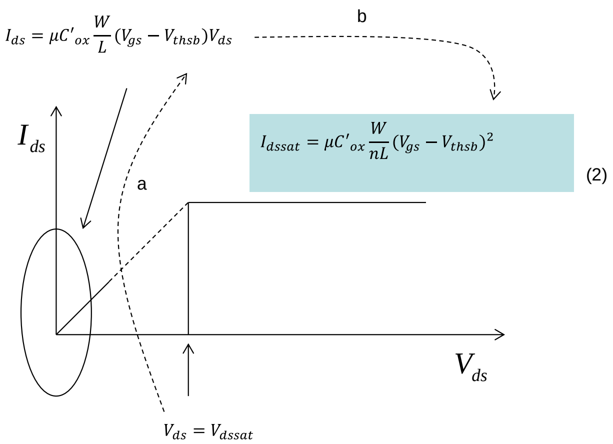


Fig 6: I_{ds} formula – simple derivation

Unfortunately, the assumption (A1) is not quite correct: the current increase is smaller Vds > ~100mV than expected from formula (2).

More precise calculation leads to an additional factor 1/2. The formula for saturation current is as follows:

$$I_{dssat} = \frac{1}{2n}\mu C'_{ox}\frac{W}{L}\left(V_{gs} - V_{thsb}\right)^2 \quad (5)$$

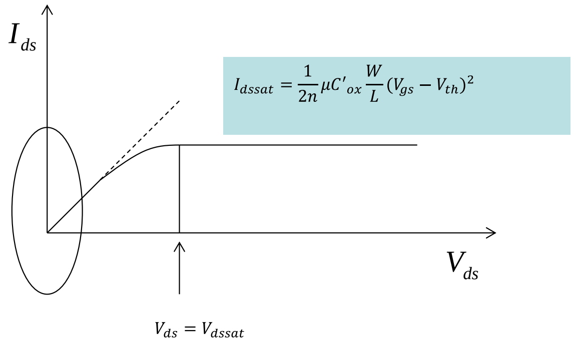


Fig 7: *I_{ds} more precise formula*

Derivation:

The following formula holds fort the channel charge per area:

$$Q' = C'_{ox}\left(V_{gs} - V_{thsb}\right) \qquad (A1)$$

This formula holds for strong inversion and small V_{ds}.

The formula must be adjusted for large V_{ds}. The channel charge per area changes from the value given by (A1) at the source side of the channel till zero at the drain side, because the channel is pinched off there. Let us denote the potential at the interface between silicon and silicon dioxide with $V_x(x)$. Coordinate x has the direction from source to drain. We define V_x with reference to V_s. This means:

$$V_x(0) = 0$$

and

$$V_x(L) = V_{ds}$$

The channel charge in the point x is:

$$Q'(x) = C_{ox}\left(V_{gx} - V_{thxb}\right) = C_{ox}\left(V_{gs} - V_{thsb} - nV_x\right) \qquad (A2)$$

or simplified

$$Q'(x) = C_{ox}\left(V_{gst} - nV_x\right);\; V_{gst} \equiv V_{gs} - V_{thsb} \qquad (A3)$$

Let us derive I_{ds} for any V_{ds}.

We start from the equation for drift current:

$$I_{ds} = \mu\, W\, Q'(x)\, |E_x| = \mu\, C'_{ox}\, W \left(V_{gst} - nV_x\right) |E_x| \qquad (A4)$$

E_x is the E-field component in x direction, W is the gate width, μ is the charge carrier mobility.

It holds:

$$|E_x| = \frac{dV_x}{dx} \qquad (A5)$$

By substituting (A5) in (A4), we obtain:

$$I_{ds} = \mu\, C'_{ox}\, W \left(V_{gst} - nV_x\right) \frac{dV_x}{dx}$$

or

$$I_{ds} dx = \mu\, C'_{ox}\, W \left(V_{gst} - nV_x\right) dv_x \qquad (A6)$$

We can integrate both sides:

$$\int_0^L I_{ds} dx = \int_0^{V_{ds}} \mu\, C'_{ox}\, W \left(V_{gst} - nV_x\right) dv_x$$

It follows:

$$I_{ds} = \frac{1}{2n} \mu\, C'_{ox} \frac{W}{L} \left[{V_{gst}}^2 - \left(V_{gst} - nV_{ds}\right)^2\right]$$

When we calculate the square, we obtain the general equation for drain source current:

$$I_{ds} = \mu\, C'_{ox} \frac{W}{L} \left[V_{gst} V_{ds} - n \frac{V_{ds}^2}{2}\right] \qquad (A7)$$

In saturation it holds:

$$V_{ds} = V_{dssat} = \frac{V_{gst}}{n} \qquad (A8)$$

When we substitute A8 in A7, we obtain the formula for the saturation current:

$$I_{dssat} = \frac{1}{2n} \mu\, C'_{ox} \frac{W}{L} V_{gst}^2 \qquad (A9)$$

Also, the formula (A9) / (5) does not describe the measurement results perfectly. More precise models show that the factor ½ can to be replaced by 1/(2nα). For V_{dssat} holds:

$$V_{dssat} = \frac{V_{gs} - V_{th}}{n\alpha} \qquad (6)$$

$$\alpha = 1 + \frac{V_{gs}}{nE_{sat}L} \qquad (7)$$

Factor α is for long transistors (transistors with L > 1µm) ~1.

For very short transistors or for large gate source voltages α is significantly larger than 1 and leads to much lower I_{dssat} values than expected. It is an effect of the saturation of mobility. In general, smaller transistors need more complex formulas.

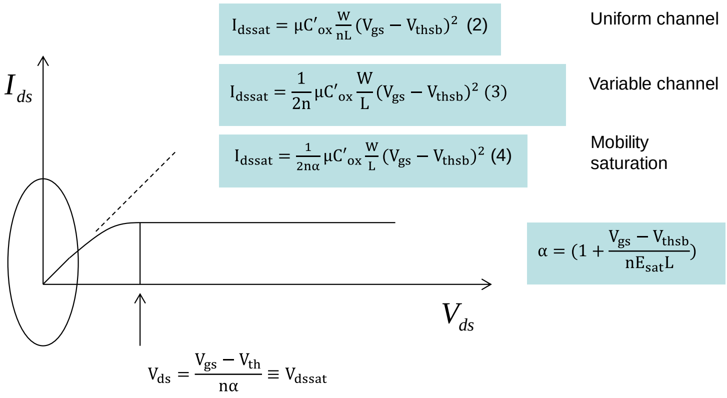
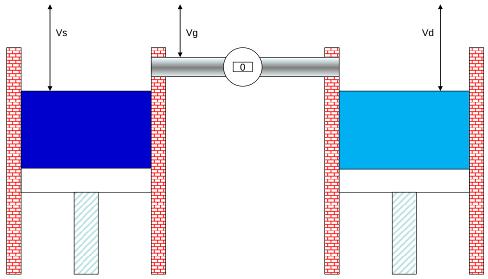
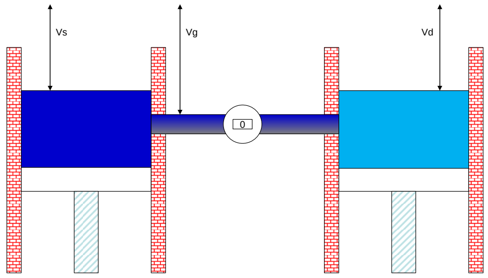


Fig 8: I_{ds} formulas – overview

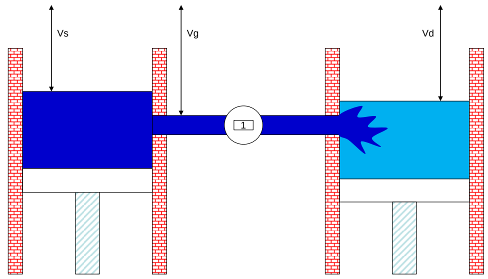
Following figures show an analogy for strong inversion.



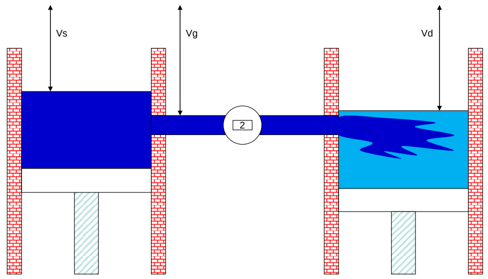
1. Imagine two water tanks (representing the ***source*** *and* ***drain****) whose bottom heights (potential energies) can be adjusted. A pipe connects the two tanks at a height, corresponding to the* ***gate voltage****.*



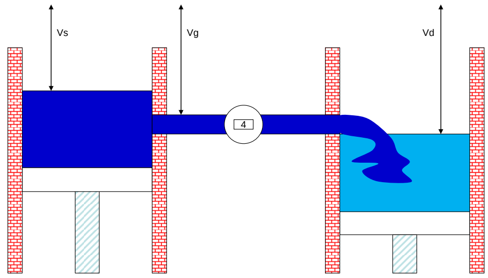
2. When the pipe lies below the water level of the source (analogous to $V_{gs}>V_{th}$), water can flow.
If the bottom heights and water levels of both tanks are the same ($V_s=V_d$), the pressure at both ends of the pipe is equal, and there is no water flow ($I_{ds}=0$).



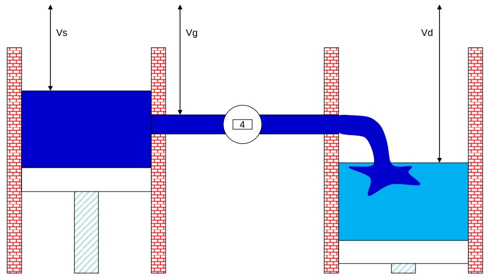
3. When the bottom of the drain tank is lowered ($V_{ds}>0$), water flows through the pipe.



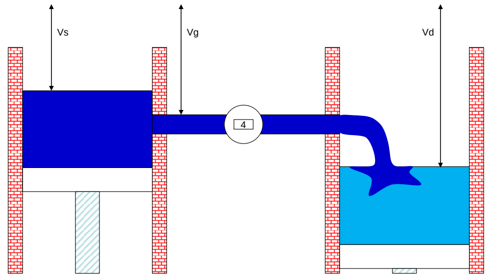
4. If the bottom of the drain tank is lowered further, the water flow increases.



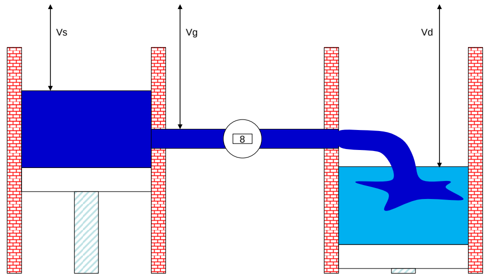
5. Now, when the water level in the drain tank is at the same height as the pipe, this corresponds to the case $V_{gd}=V_{th}$ — the ***beginning of saturation****. The water flow reaches its maximum.*



6. If the water level in the drain tank becomes lower than the pipe ($V_{gd}<V_{th}$), the flow no longer changes — the ***channel is pinched off****.*



7. If the water level in the drain tank becomes lower than the pipe ($V_{gd}<V_{th}$), the flow no longer changes — the ***channel is pinched off****.*



8. If we lower the pipe further (increase V_{gs}), more water flows.

MOSFET equations

In the following text, we assume $V_s = V_b = 0V$.

The electrical state of the transistor is described by two voltages, V_{gs} and V_{ds}, and by two currents I_{ds} and I_{gs}. For DC signals, $I_{gs} = 0$ holds. Gate is just a capacitance. (We are neglecting the gate current due to tunnel effect.)

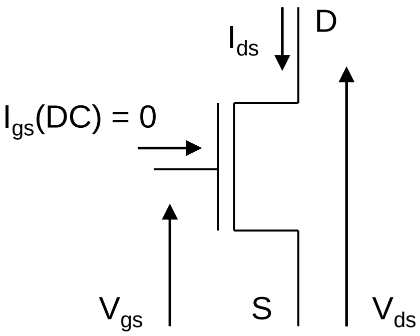
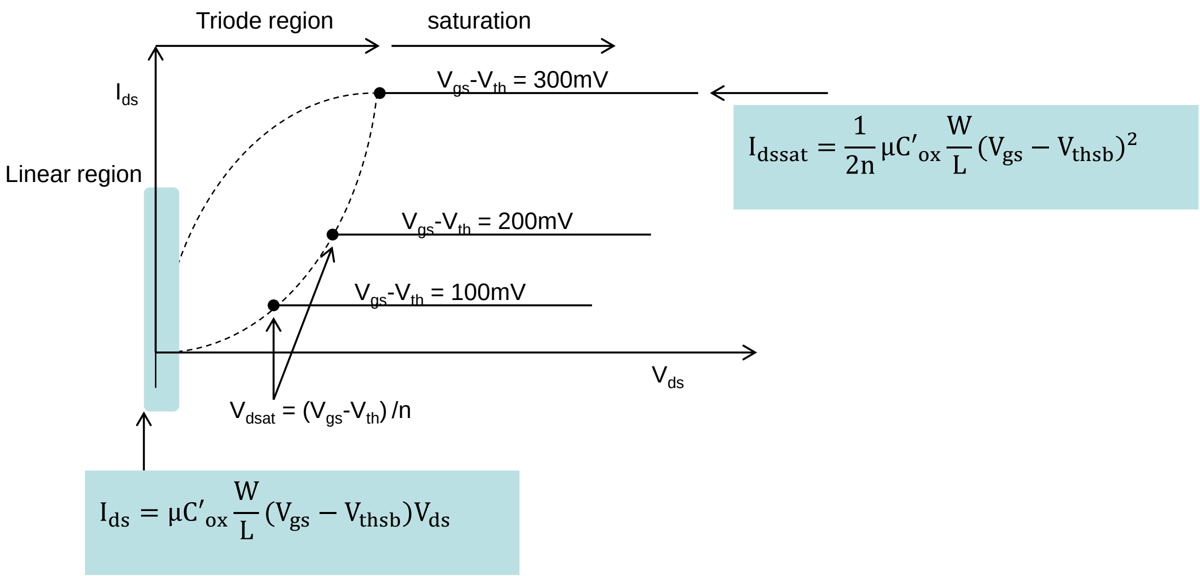


Fig 9: Transistor

Fig 9 shows the transistor I-U characteristics: Transistor behavior for DC signals can be described with the following characteristics. (DC-Signals = slow voltages and currents of any amplitude)



$$I_{dssat} = \frac{1}{2n}\mu C'_{ox}\frac{W}{L}(V_{gs} - V_{thsb})^2$$

$$I_{ds} = \mu C'_{ox}\frac{W}{L}(V_{gs} - V_{thsb})V_{ds}$$

Fig 10: Output characteristics of MOSFET

I_{ds} as a function of V_{ds} for different V_{gs} (output characteristics)

I_{ds} as a function of V_{gs} for different V_{ds} (input characteristics)

Let us consider the output characteristics.

Fig 10 shows the I_{ds} - V_{ds} characteristics. The lines are plotted for linearly increased V_{gs}, e.g. 100 mV, 200 mV...

In the right line regions, the current is nearly independent of V_{ds}. We call this region the saturation region. Ideally $I_{ds} = I_{dssat}$ for $V_{ds} > V_{dssat}$. The saturation voltage is nearly

$$V_{dssat} = \frac{(V_{gs} - V_{thsb})}{n}$$

The saturation current I_{dssat} is given by the following formula:

$$I_{dssat} = \frac{1}{2n} \mu C'_{ox} \frac{W}{L} \left(V_{gs} - V_{thsb}\right)^2 \quad (8)$$

In the left line regions, the current decreases with the decrease of V_{ds}. We call is triode region. For small V_{ds}, the current voltage is approximately linear function if V_{ds} (linear region). The current in the linear region can be described with the following formula:

$$I_{dssat} = \mu C'_{ox} \frac{W}{L} \left(V_{gs} - V_{thsb}\right) V_{ds}$$

The current in saturation (8) depends quadratically of V_{gs}. $V_{ds} = V_{dssat}$ at the transition between the saturation and triode regions.

It can be shown that (I_{ds}, V_{ds})-points between the triode and the saturation regions are located at the function $I_{ds} = k/2\ V_{ds}^2$.

This can be derived from the formula for saturation current (8) and the condition

$$V_{ds} = V_{dssat} = \frac{(V_{gs} - V_{thsb})}{n}$$

In the saturation region, the transistor acts like a voltage-controlled current source. Such current sources are useful. It is possible to implement a voltage amplifier having a high voltage gain using a current source and a large resistance.

In the triode region for small V_{ds}, the transistor acts as a variable resistance.

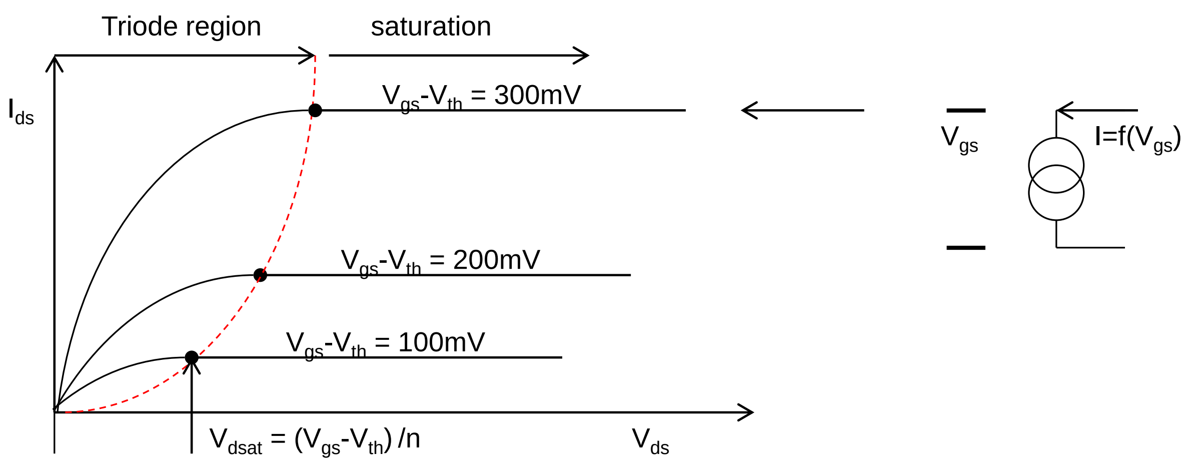


Fig 11: Output characteristice, saturation

Fig 12 shows the characteristic of I_{ds} - V_{gs}. We show in this figure only the currents at the beginning of the saturation: I_{dssat} as function of V_{gs}.

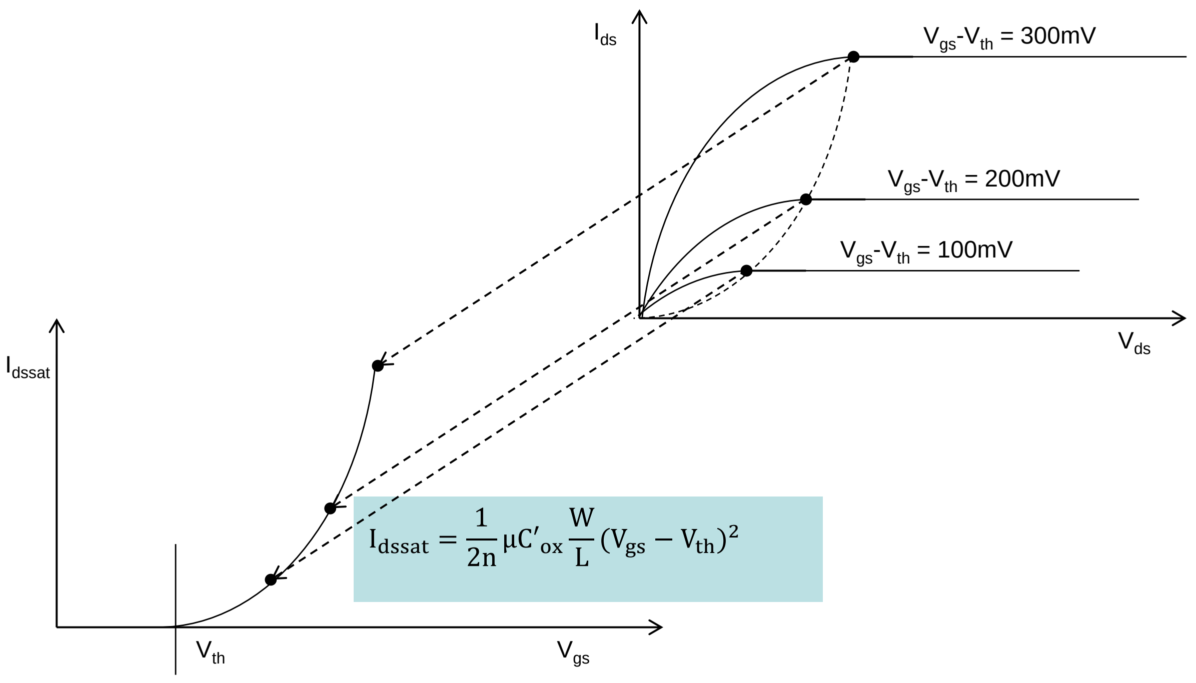


Fig 12: Input characteristics

Small signal model:

The input characteristics is usually linearized in the region around the operation point. The slope dI_{dsat}/dV_{gs} is called the **transconductance** (g_m), Fig 13.

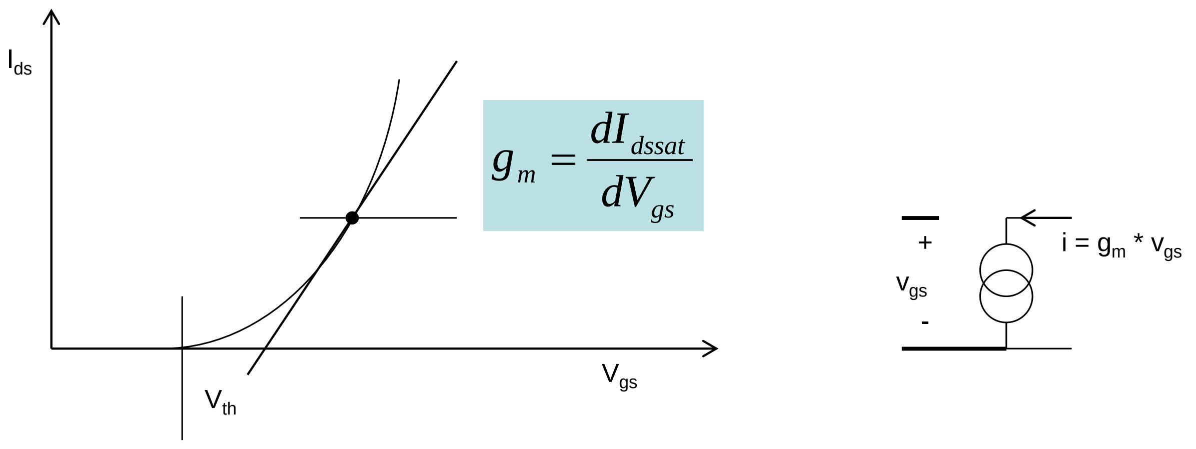


Fig 13: Transconductance

Using g_m as parameter we can derive the small signal model of a MOSFET shown in **Fehler! Verweisquelle konnte nicht gefunden werden.**, right.

Notice that the small signal models are valid under certain conditions. The small signal model allows mathematically a large positive and negative v_{gs} and i_{ds} values (small signals). However, the negative small signal current must not exceed the DC current, otherwise the total current would be negative.

It holds:

$$g_m = \frac{dI_{dssat}}{dV_{gs}}$$

From the formula for saturation current:

$$I_{dssat} = \frac{1}{2n} \mu C'_{ox} \frac{W}{L} (V_{gs} - V_{thsb})^2$$

we get the formula for transconductance in strong inversion:

$$g_m = \sqrt{2kI_{dssat} \cdot (W/L)}$$

with

$$k = \frac{1}{n} \mu C'_{ox}$$

Differences between PMOS und NMOS

In the case of the PMOS, the I-V characteristics lines are equal as in the case of the NMOS if the signs of voltages and currents are changed, Fig 14.

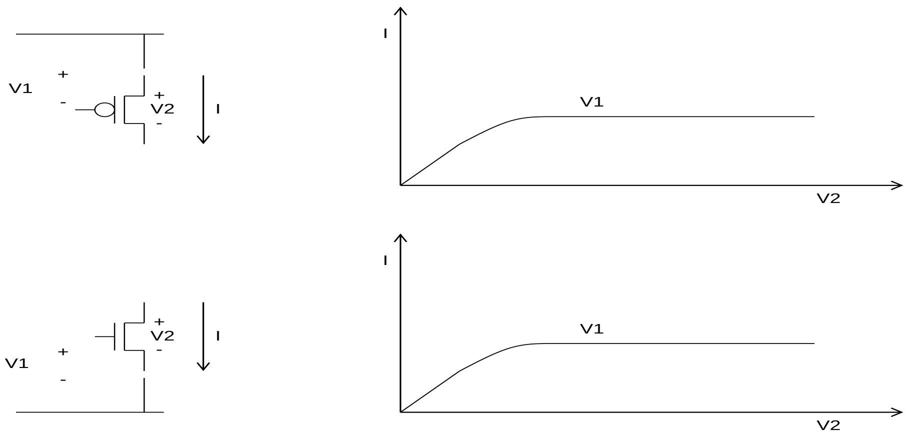


Fig 14: I-V characetristics of PMOS (top) and NMOS (bottom)

PMOS circuits are often a mirror image of NMOS circuits in the way the Fig 15 shows.

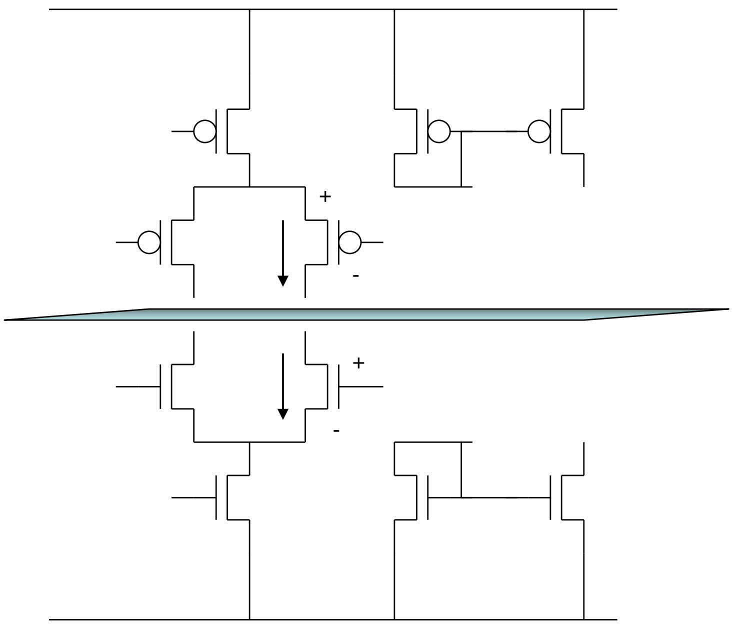


Fig 15: PMOS and NMOS circuits are often symmetrical

The currents and voltages have opposite signs. We will draw circuits in the way that the currents flow from top to bottom and the potentials above in the image are higher than the potentials below.

It is important to determine the operation region (triode-, saturation-region) for every transistor. In analogue circuits, transistors operating is saturation are especially useful. The condition for saturation is $V_{ds} > (V_{gs} - V_{th})/n$. This means for an NMOS that the drain potential may be lower than the gate potential. Fig 16 and Fig 17show transistors that work in saturation and in linear region.

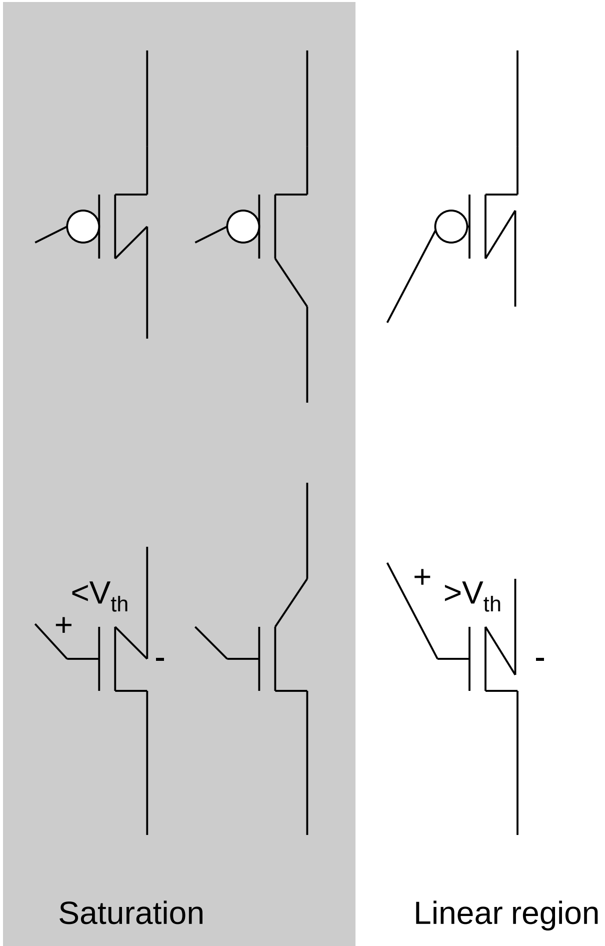


Fig 16: NMOS and PMOS transistors in saturation and linear region. The position of a node or line illustrates its potential. If the position is low, the potential is low as well.

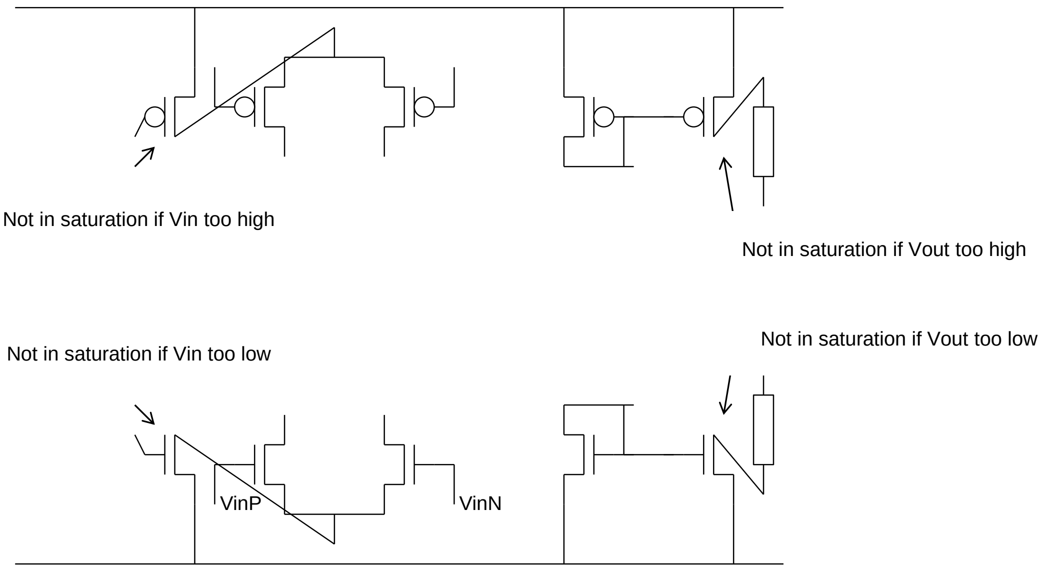


Fig 17: NMOS and PMOS circuits

Why do we need NMOS and PMOS transistors?

An NMOS transistor conducts current, only when its source potential is low. This means, for instance, that an NMOS-based electronic switch cannot be used to short a circuit with VDD (with positive supply). This is one of the reasons why we need PMOS transistors.

Example: A capacitor is discharged with an NMOS transistor T1, Fig 18. The capacitor is then charged with another NMOS transistor T2. The voltage at the capacitor does not reach VDD. When V_{gs} of T2 becomes lower than V_{th}, the transistor becomes off and it cannot charge the capacitor further. (We neglect the subthreshold current.)

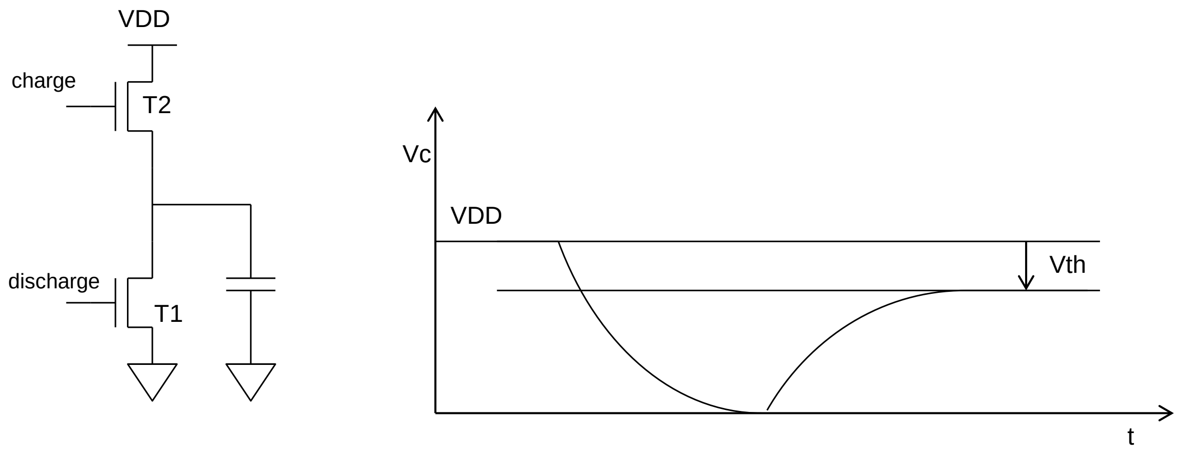


Fig 18: A capacitor is discharged and charged with NMOS transistors.

If the capacitor is charged with PMOS transistor, as Fig 19 shows, the capacitor voltage reaches VDD quickly.

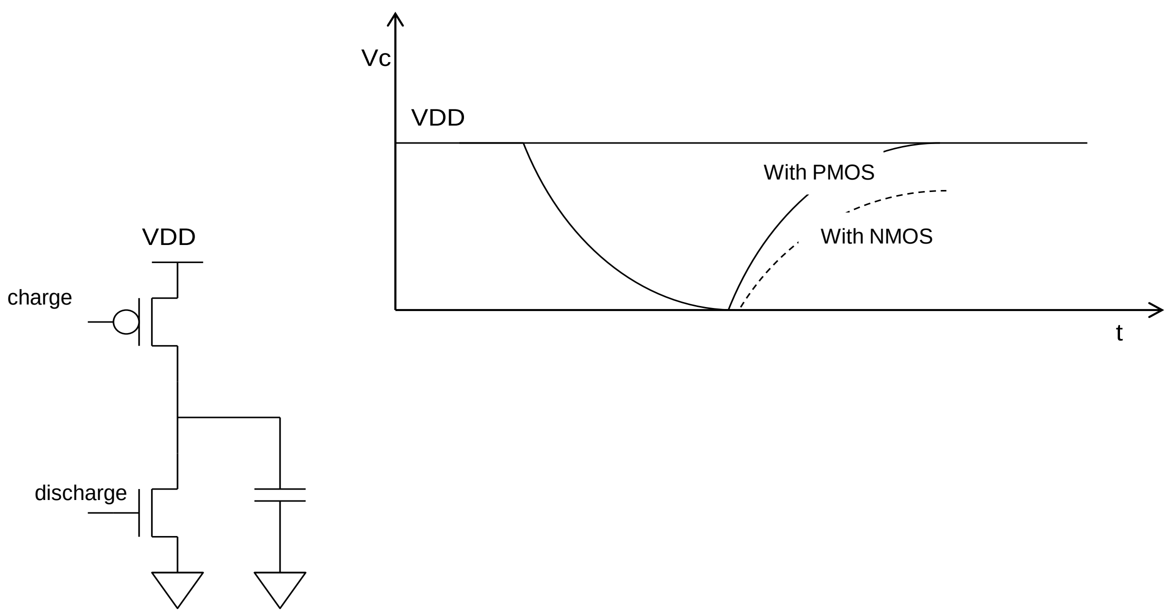


Fig 19: A capacitor is discharged with NMOS transistor and charged with PMOS transistor.

There are several differences when NMOS and PMOS transistors are used. For instance, in the case of a PMOS current source, Fig 20 right, the current flows out of VDD. An NMOS source conducts the current (drains the current) to GND, Fig 20 left.

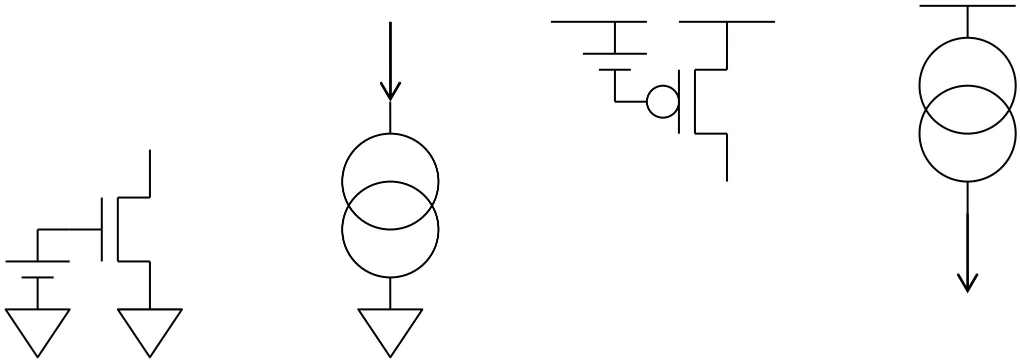


Fig 20: Current sources made with NMOS and PMOS transistors

Symmetrical Formula for triode- and saturation region (optionally)

We have derived the following formula that holds in triode region

$$I_{ds} = \mu C'_{ox} \frac{W}{L}\left((V_{gs} - V_{thsb})V_{ds} - n\frac{{V_{ds}}^2}{2} \right); V_{thsb} \equiv V_{th} + (n-1)V_{sb} \quad (B1)$$

The formula is not very elegant because it is asymmetrical with respect to source and drain. The transistor is, on the other hand, symmetrical. It would be better if the formula reflects this symmetry. This can be fixed in the following way.

$$I_{ds} = \frac{1}{2n}\mu C'_{ox}\frac{W}{L}(V_{gs} - V_{th} - (n-1)V_{sb})^2 - \frac{1}{2n}\mu C'_{ox}\frac{W}{L}(V_{gd} - V_{th} - (n-1)V_{db})^2 \quad (B2)$$

or

$$I_{ds} = \frac{1}{2n}\mu C'_{ox}\frac{W}{L}(V_{gs} - V_{thsb})^2 - \frac{1}{2n}\mu C'_{ox}\frac{W}{L}(V_{gd} - V_{thdb})^2 \quad (B3)$$

It can be shown that formulas B2 and B3 follow from (B1) when the terms in brackets are multiplied.

The formula B3 gives us the idea to represent a transistor as a parallel circuit of two ideal transistors. These ideal transistors conduct current only from drain to source and are always in saturation. Such a representation is useful to solve certain circuits.

The equation B3 can be written as follows:

$$I_{ds} = I_{dssat}(V_{gs}, V_{thsb}) - I_{dssat}(V_{gd}, V_{thdb})$$

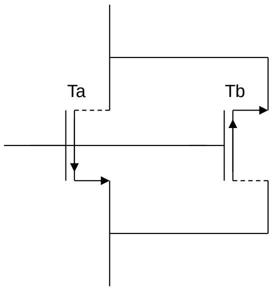


Fig 21: General transistor model

We can for instance understand the current saturation in alternative way. The current saturates when Tb does not conduct any more.

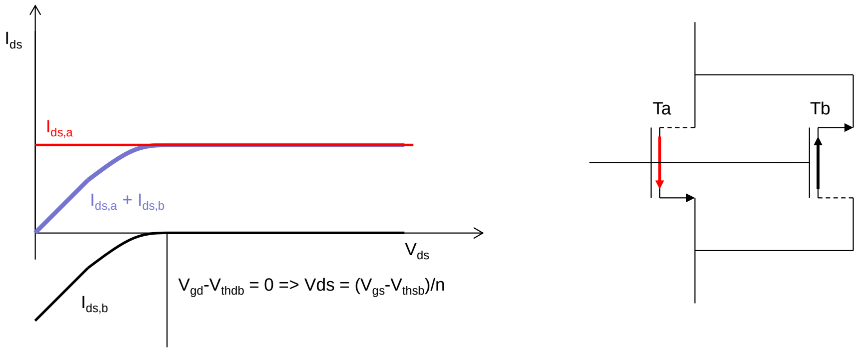


Fig 22: General transistor model – saturation of the current