

Lecture 6

This lecture covers following themes:

Summary of the MOSFET equations

Summary and proof of Mason's and Blackmans's formulas for calculation of voltage gain and impedances of an amplifier with feedback

Basics of frequency analysis, Bode plot, step response, high pass and low pass filter

Frequency analysis and step response of the inverting voltage amplifier

Method for fast estimation of coefficients in characteristic polynomial

Brief summary of MOSFET equations

In lectures 1 and 2 we have described MOS transistors, their production and functioning. We have derived the formulas for drain-source current as a function of V_{gs} and V_{ds} .

The currents are represented as input and output characteristics.

One important parameter is the threshold voltage (V_{th}). V_{th} was calculated in our example to be about 0.5 V.

For V_{ds} voltages higher than $V_{dssat} \sim V_{gs} - V_{th}$, the drain current saturates, Fig 2.

The value V_{dssat} is called the saturation voltage.

When we took body effect into account we derived

$$V_{dssat} \sim V_{gs} - V_{th}/n.$$

N is the subthreshold factor $n = 1 + C_{dep,min}/C_{ox} \sim 1.25$.

All equations we have derived for NMOS also apply to PMOS transistors when we negate voltages (V_{gs} and V_{ds}) and currents (I_{ds}) or when we use absolute values instead of signed values for I and V.

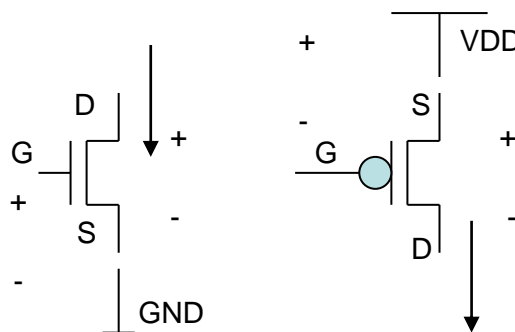


Fig 1: NMOS and PMOS

The current in the saturation region is:

$$I_{dssat} = \frac{1}{2n} \mu C_{ox} \frac{W}{L} (V_{gs} - V_{thsb})^2$$

$$V_{thsb} \sim V_{th} + (n-1) V_{sb}$$

For a 65 nm technology:

E_{sat} for PMOS ~ 10.4 V/ μ m and E_{sat} for NMOS ~ 9.7 V/ μ m

μ (NMOS) = 2.64×10^{-2} m²/Vs and μ (PMOS) = 1.45×10^{-2} m²/Vs

C'_{ox} = 13.28 fF/ μ m² or 0.01328f/m²

$V_{th} \sim 0.4$ V („regular“ V_{th} Transistors)

For $V_{ds} > V_{dssat}$ the current increases only slightly due to the channel length modulation (Early Effect).

It holds:

$$r_{ds} = \frac{1}{g_{ds}} = \frac{E_{sat} L}{I_{dssat}}$$

R_{ds} is 1/current increase, E_{sat} is a constant, L is the transistor gate length.

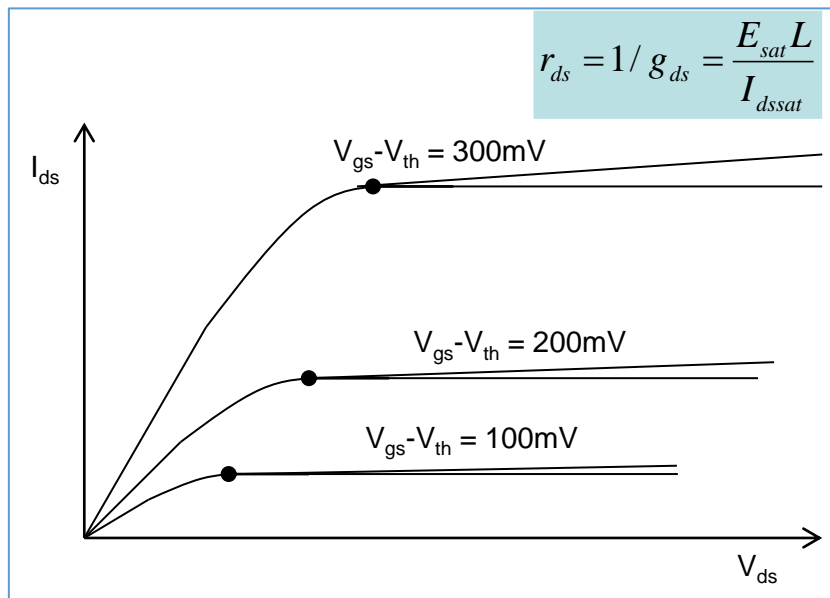


Fig 2: Output characteristics of a MOSFET

For small signals, the transistor in saturation behaves as a voltage controlled current source (Fig 4) with the following transconductance:

$$g_m = \frac{dI_{dssat}}{dV_{gs}}$$

And the output resistance r_{ds} .

It can be derived:

$$g_m = \frac{dI_{dssat}}{dV_{gs}} = \frac{2I_{dssat}}{(V_{gs} - V_{thsb})} = \frac{2I_{dssat}}{nV_{dssat}}$$

The gate capacitance is $C_{gs} = WL C_{ox}$.

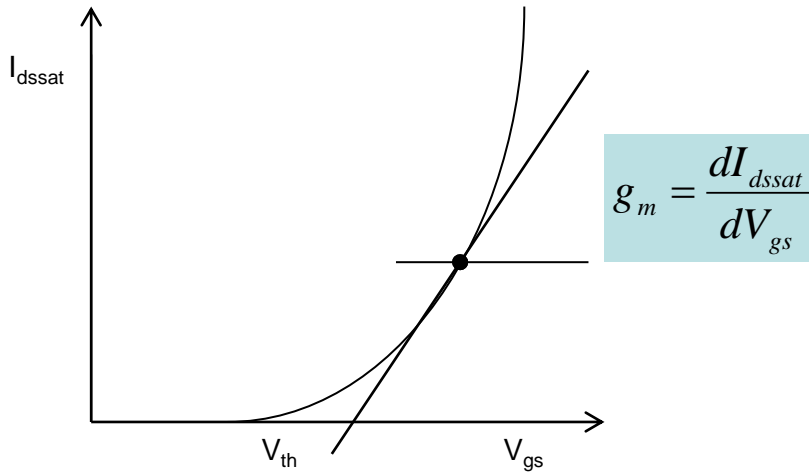


Fig 3: Input characteristic of a MOSFET

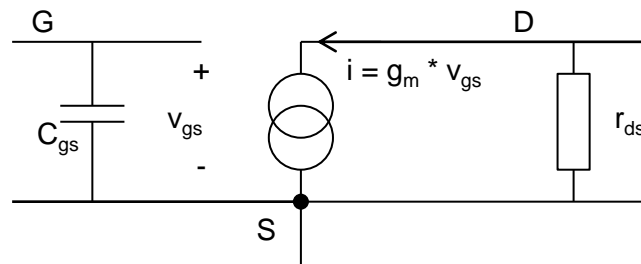


Fig 4: Small signal model of a MOSFET

Summary: DC-, AC-analysis, Mason's gain formula and Blackman's formula for calculation of impedances

An important circuit is the voltage amplifier with a transistor (T_{in}), a resistor (R_l) and the feedback circuit C_i , C_f , R_f , shown in Fig 5.

We analyse the inverting amplifier in the following way:

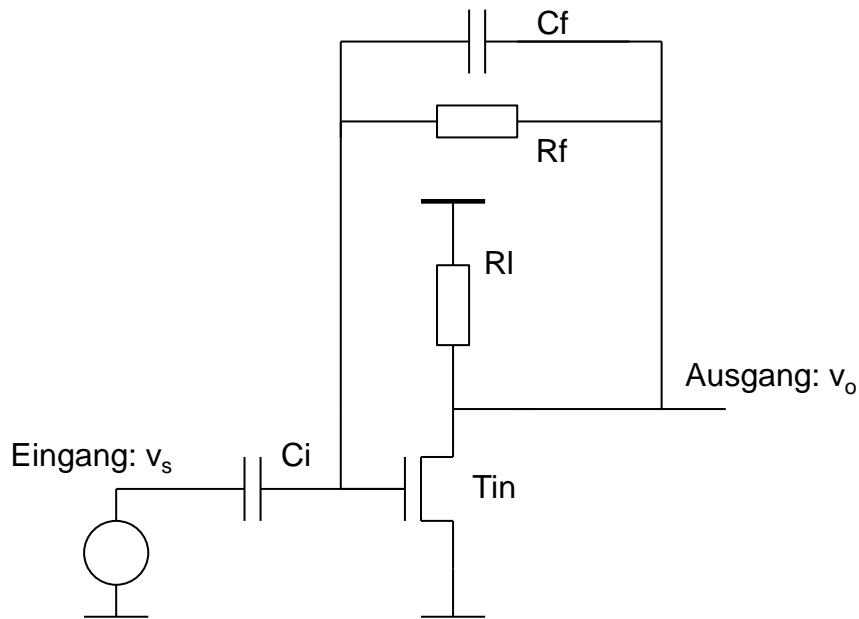


Fig 5: Inverted amplifier with feedback

We assume that the voltages and currents have a constant mean value (DC value) and a smaller time variable signal (AC value).

In that case, we can perform the DC and AC analyses separately, as illustrated in Fig 6.

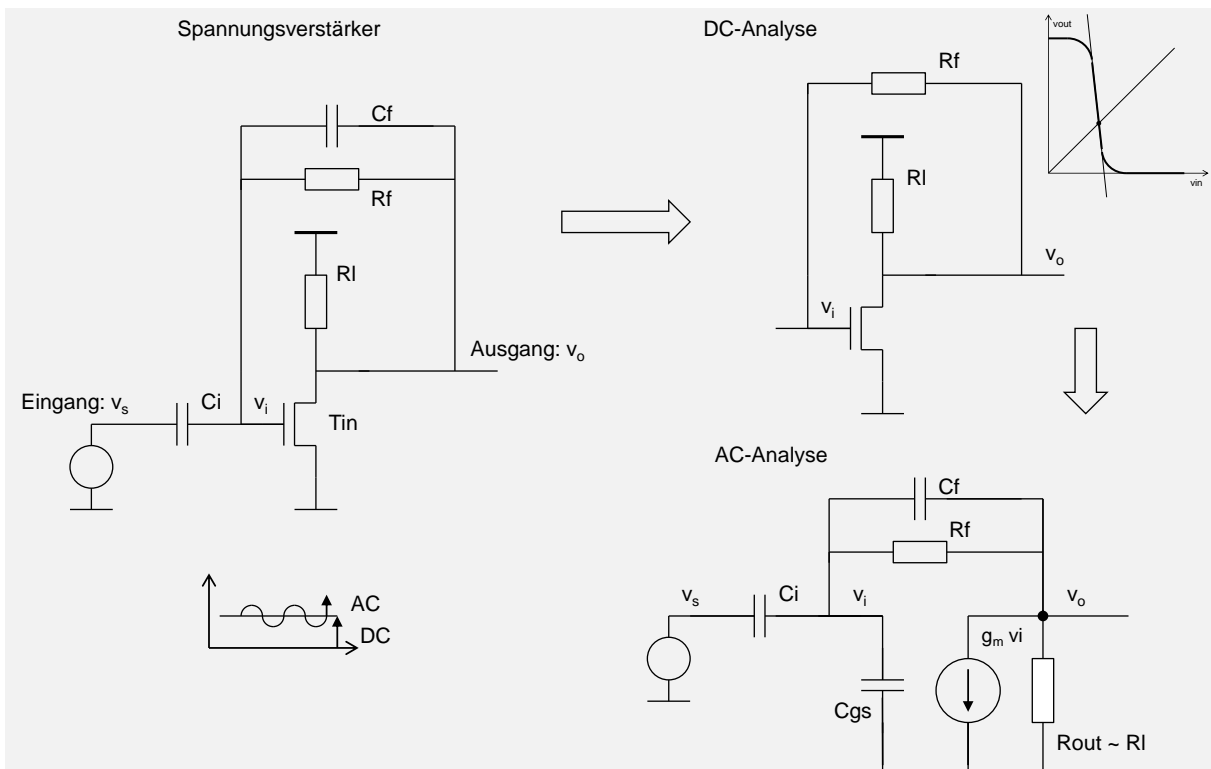


Fig 6: AC and DC analysis of the inverted amplifier

DC analysis

DC values remain constant for a long time. Therefore, capacities and inductivities can be removed during the DC analysis. The components are described by non-linear equations (or by

characteristics). The DC voltages of transistor nodes are called working points (or operating points).

AC analysis

The $I = f(V)$ equations of the components are linearized in the working point, which leads to small signal models.

The small signal model of a MOSFET is a linear amplifier – voltage controlled current source.

A small signal model of a constant voltage source is the ground potential (mass).

In this way we get linear circuits with the resistors, inductivities, capacities and linear amplifiers.

If the circuit has one input and one output, it can be described by a linear differential equation or by a complex transfer function.

Ideally, an amplifier should only amplify the AC signal and keep the DC value unchanged, because DC value carries not useful information.

This behavior can be achieved with appropriate feedback.

In our example we have a DC and an AC feedback, Fig 6.

DC feedback is implemented using a resistor (R_f) or as a switch. Capacitance C_f provides the AC feedback.

The DC analysis can be carried out graphically, the characteristics of the amplifier and the DC feedback are drawn on the same graph. The working point is at the intersection point.

AC analysis is performed on the small signal circuit.

Mason's formula

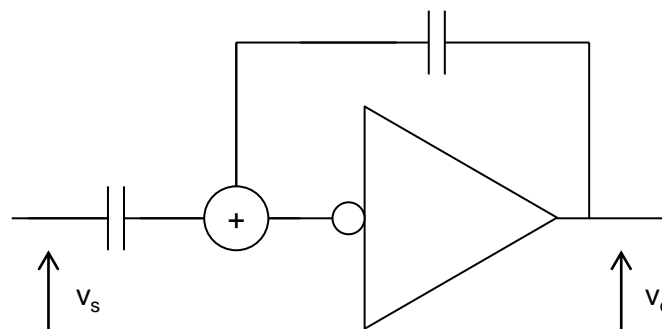


Fig 7: Small signal model of the inverted amplifier

The AC model of voltage amplifier is a linear circuit with an input v_s , an output v_o , an amplifier and feedback, as shown in Fig 7.

Let us calculate $v_o = f(v_s)$.

We will solve the circuit using a trick:

The line at the input of the amplifier is cut (Fig 8). The cutting point should be chosen in the way that the current through the line that is cut was zero (before cutting).

An additional voltage source (a second input) v_i^* is added.

An additional output v_i is defined.

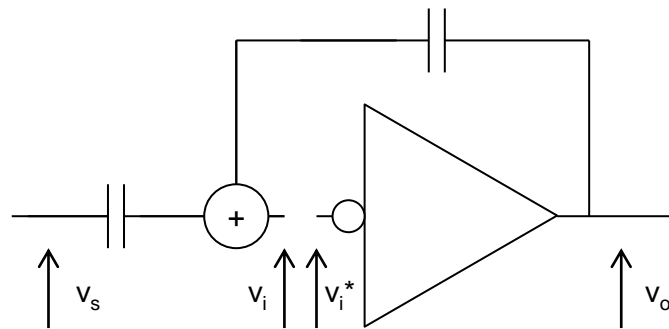


Fig 8: The feedback is cut

This way we get from the original circuit two circuits that are easier to solve, as the signals only run in one direction. This is illustrated in Fig 9.

These circuits can be described by the following equations:

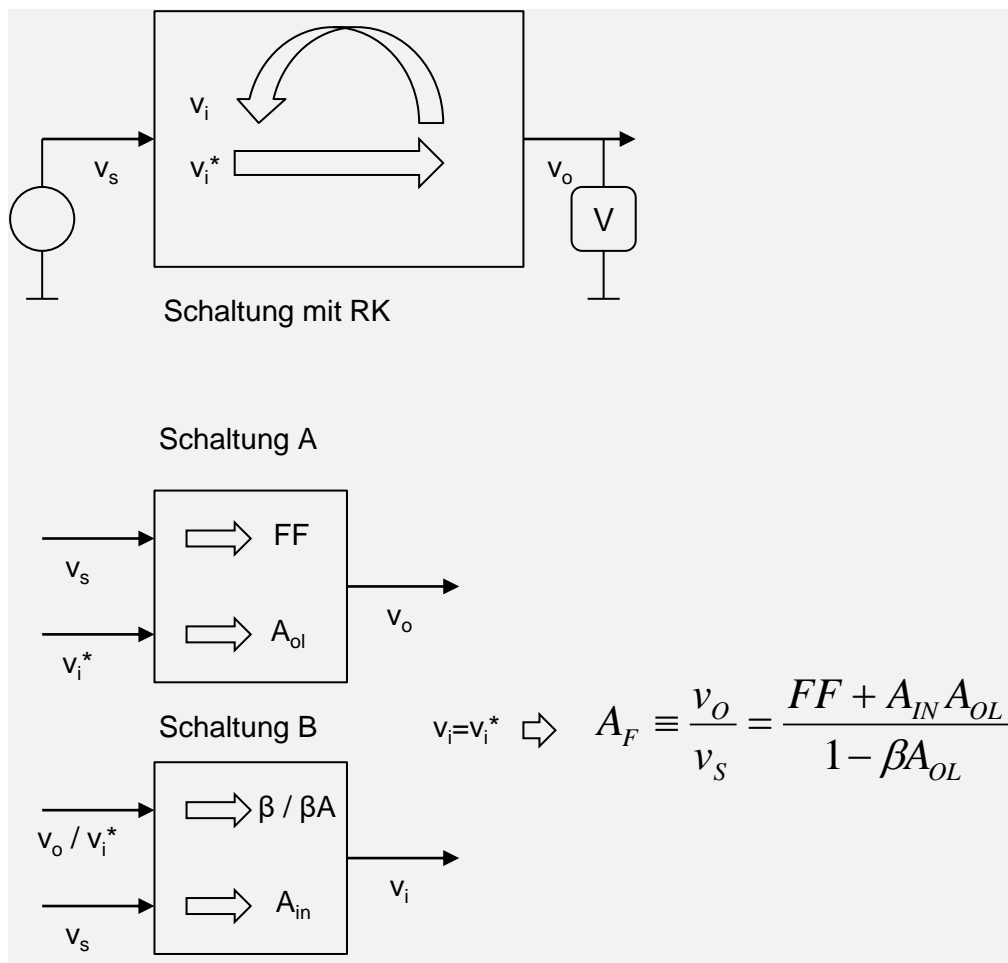


Fig 9: The original circuit with feedback is split into two circuits without feedback

Circuit A:

$$v_o = FFv_s + A_{ol}v_i *$$

Circuit B:

$$v_{in} = A_{in}v_s + \beta v_o$$

The coefficients have the following names

FF – feed forward

A_{ol} – open loop gain

A_{in} – input gain

β – feedback

If we take into account the condition $v_i = v_i^*$ we can easily derive from (1) and (2) the formula, known as Mason's gain formula, for the transfer function of the original circuit:

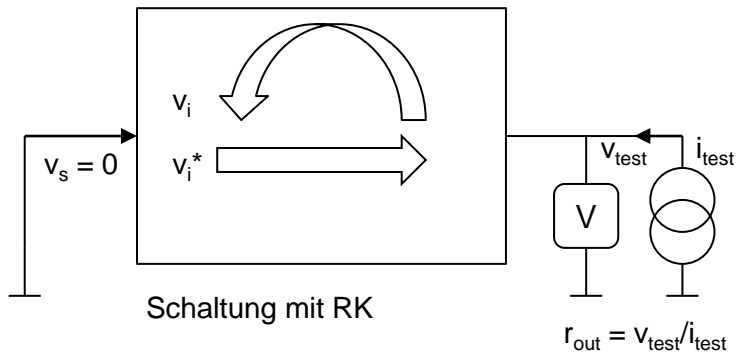
$$v_o = v_s \frac{FF + A_{in}A_{ol}}{1 - \beta A_{ol}} \quad (1)$$

Feedback has the following characteristics:

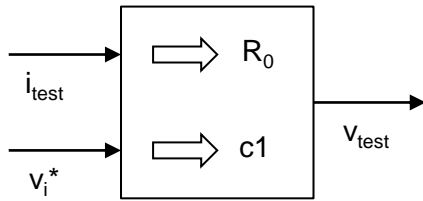
If the loop amplification βA is high, the voltage at the input of the amplifier v_i does not change. We say that the v_i node is a virtual mass. The amplification with feedback (v_o/v_s) depends in this case practically only on β .

Blackman's formula

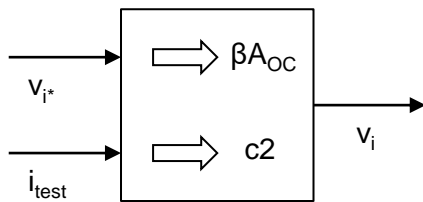
We can derive in a similar way the formula for the output impedance, or impedance between any two points. Fig 10 and Fig 11 show two test circuits for the calculation of the output resistance of the circuit with feedback R_{FB} . The first circuit uses one voltage source v_{test} . Current i_{test} is measured (calculated) in order to calculate R_{FB} . The second circuit uses a current source i_{test} while the voltage is measured. In both cases it is $R_{FB} = v_{test} / i_{test}$. Both circuits must lead to the same R_{FB} result. When the original circuit is split into circuits A and B and when the condition $v_i = v_i^*$ is taken into account, formulas A1 and A2 can be derived (Fig 10 and Fig 11). From (A1) and (A2) follows the formula A3 (Fig 11), known as Blackmans formula.



Schaltung A

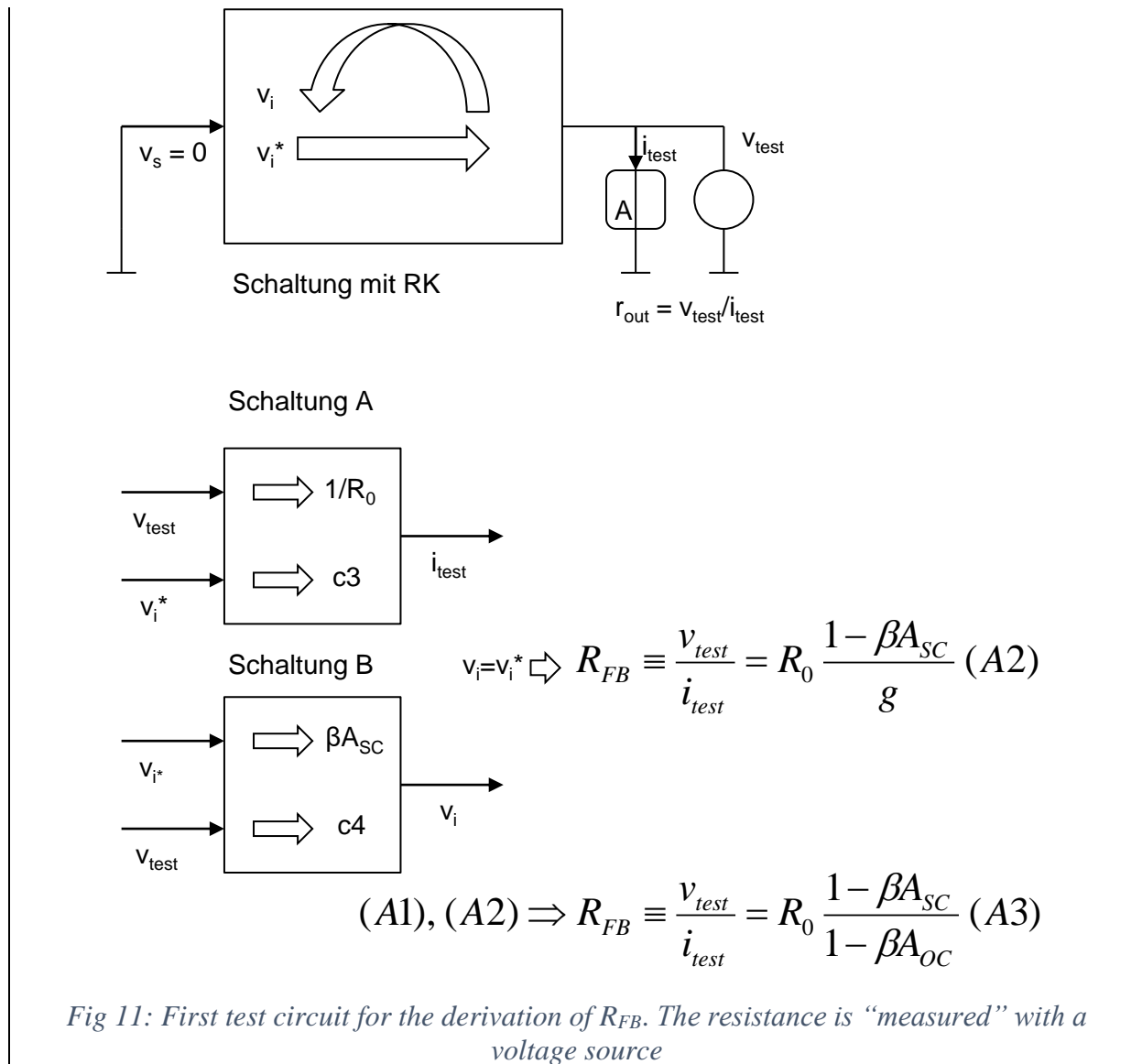


Schaltung B



$$v_i = v_i^* \Rightarrow R_{FB} \equiv \frac{v_{test}}{i_{test}} = R_0 \frac{f}{1 - \beta A_{OC}} \quad (A1)$$

Fig 10: First test circuit for the derivation of R_{FB} . The resistance is “measured” with a current source



Basics of frequency analysis, Bode plot, step response, high pass and low pass filter

The theme of this lecture are frequency-dependent circuits.

Let us first take as example of a simplest low pass filter that consists of a resistor and a capacitor, Fig 12. The output of the circuit is at the capacitor.

We can derive the complex transfer function as the ration between output and input voltage ($u_{out}(s)/u_{in}(s)$). $u_{out}(s)$ is the spectral function of the output voltage $u_{out}(t)$ obtained by Laplace Transformation. $u_{in}(s)$ is the Laplace transformation of the input voltage $u_{in}(t)$.

The transfer function can be derived when the circuit is solved by means of the node potential method, Kirchhoff rules, or otherwise. Formulas for complex impedance have to be used. These can be derived from the following differential equations for currents and voltages:

Capacitor: $i_c = C D u_c$

Inductor: $u_l = L D i_l$

Resistance: $u_r = R i_r$

Symbol D is the derivation operator $D \equiv d/dt$.

If the derivation operator D is replaced by the complex frequency s, we obtain the following Laplace transformed equations:

Capacitor: $i_c = s C u_c$

Inductor: $u_l = s L i_l$

Resistance: $u_r = R i_r$

And the corresponding impedances:

Impedance of capacitor $Z_c = 1/sC$ (2)

Impedance of inductivity $Z_l = sL$ (3)

Impedance of resistance $Z_R = R$ (4)

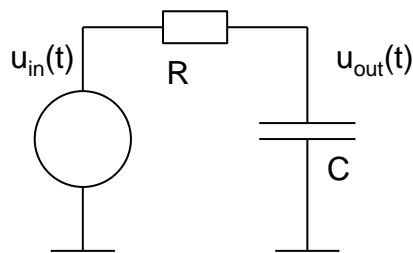


Fig 12: Low pass filter

The simplest way to find the output voltage of the low pass circuit is by using the formula for the voltage divider:

$$u_{out} = u_{in} \frac{Z_C}{Z_C + Z_R} \quad (5)$$

When we substitute the formulas for impedances (2) – (4) into (5) we obtain:

$$u_{out}(s) = u_{in}(s) \frac{1}{1 + sRC} \quad (6)$$

We can also write the derivative operator D instead of frequency s in (6) and multiply both sides with the factor in the denominator. Then we get the linear differential equation:

$$(RC D + 1)u_{out}(t) = RC \frac{du_{out}(t)}{dt} + u_{out}(t) = u_{in}(t) \quad (7)$$

U_{in} can be any function of time. A step function (or Heaviside function) is especially useful. We call the response of the circuit to the step function the step response.

Impulse response is the response to a Dirac impulse with area 1 at the input. It is not to be mixed with the step response, which is the answer to a step function at the input. Impulse response can be derived from the step response by time derivative. On the other hand, the step response is integral to the impulse response. Impulse response can also be derived from the transfer function $V_{out}(s) / V_{in}(s)$ by inverse Laplace or Fourier transformation because the Laplace or Fourier transformations of a Dirac impulse are 1.

Why is the step response important?

1. Step response can be easily intuitively derived.
2. From the step response $u_{out}^*(t)$ we can calculate the response to any time dependent function $f(t)$ by convolution of $u_{out}^*(t)$ and $f(t)$, Fig 13.
3. In this course we analyse also the switched circuits and their input signals have the shape of step function

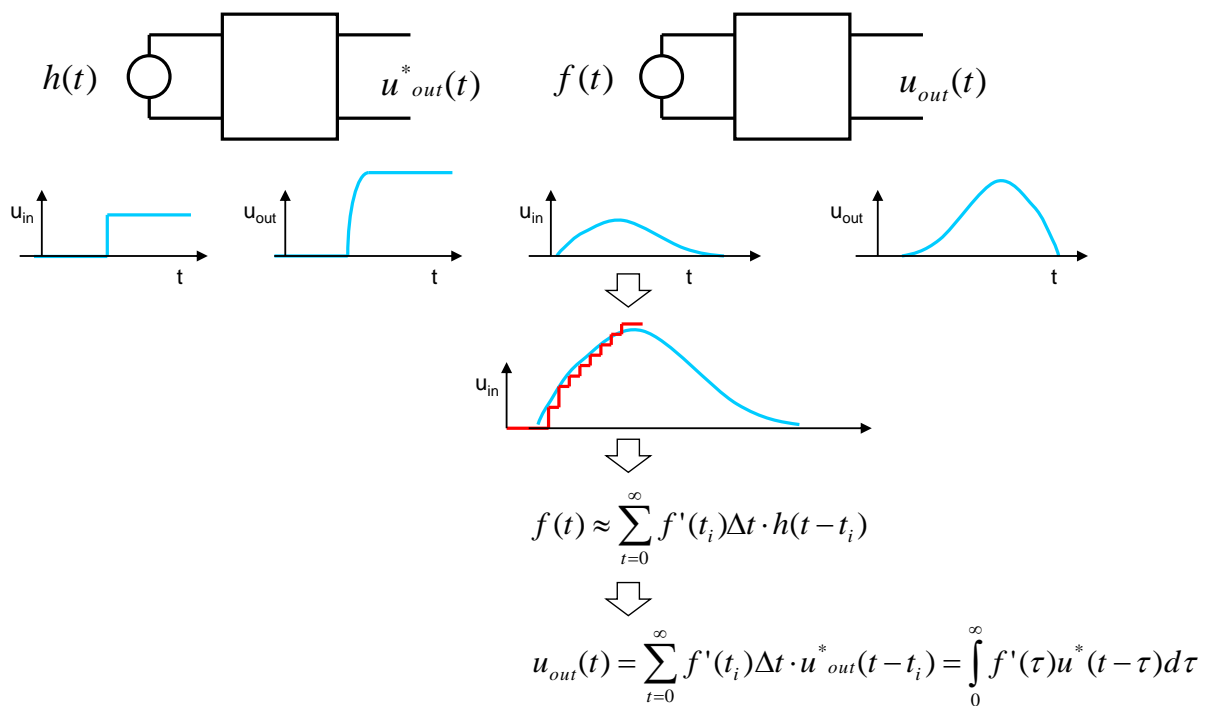


Fig 13: Response to a function $f(t)$

Let us now calculate the step response of the low pass circuit from Fig 12.

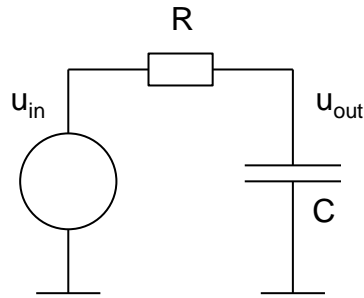


Fig 14: Low pass filter

As mentioned above, we can derive the linear differential equation from the complex transfer function if we replace complex frequency s with operator D . We obtain:

$$(RC D + 1)u_{\text{out}}(t) = h(t) \quad (8)$$

$H(t)$ is the step function with amplitude 1, referred to as the Heaviside function.

We will not solve the equation completely then discuss the behaviour of the circuit shortly after we “switch on” the input voltage (for short time scale) and after longer time has passed, so that the voltages practically does not change anymore (for long time scale).

When we consider the long time scale, the terms with time derivatives can be neglected as voltages do not change. The equation (8) is reduced to:

$$u_{\text{out}}(\infty) = 1$$

For short time scale ($t \ll \text{time constant } RC$), the terms without derivative can be neglected. The equation (8) gets reduced to:

$$RC D u_{\text{out}}(t) = h(t)$$

or

$$u_{\text{out}}(t) = \frac{1}{RC} \frac{h(t)}{D} = \frac{1}{RC} \int h(t) dt = \frac{t}{RC}$$

This is illustrated in Fig 15.

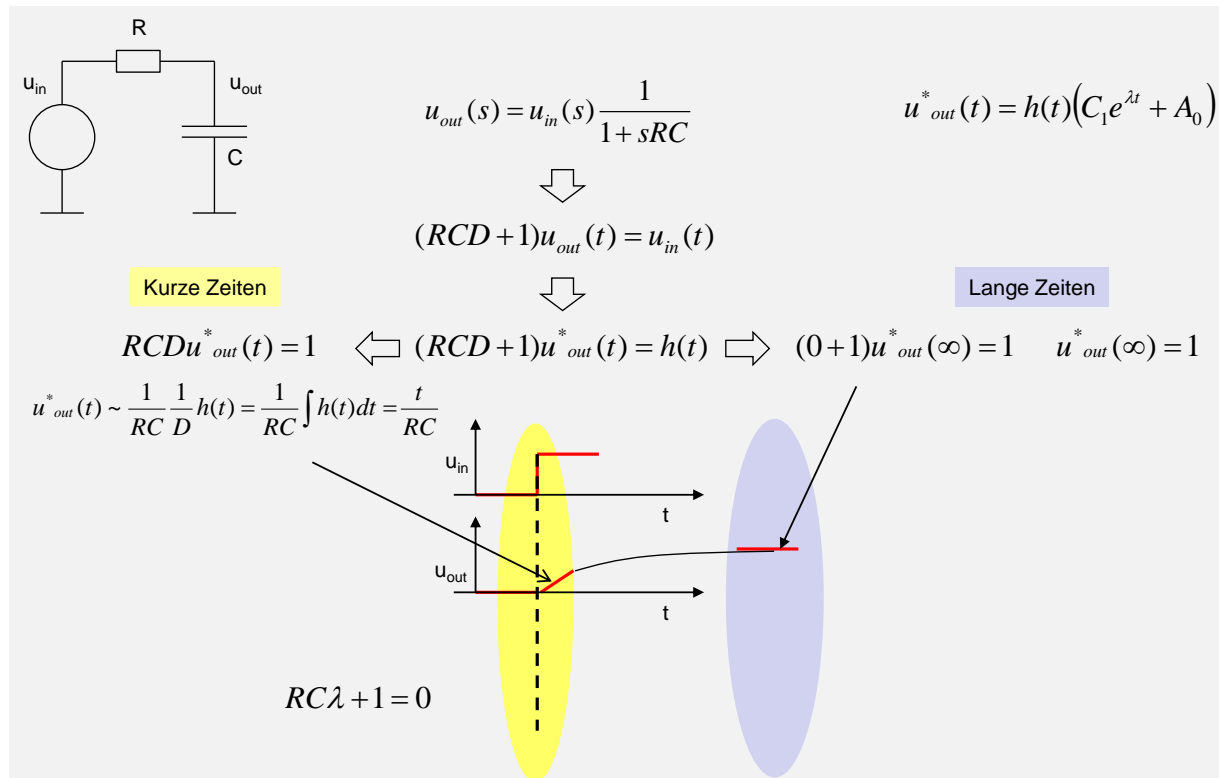


Fig 15: Solutions of $u_{out}(t)$ for short and long time scales

For intermediate times, we have to solve the differential equation. The solution has the following form:

$$A = C_1 e^{\lambda t} + A_0$$

Lambda is the solution of the following characteristic equation:

$$RC\lambda + 1 = 0$$

It holds:

$$\lambda = -\frac{1}{RC}$$

RC is the time constant.

If we adjust the constants C_1 and A_0 so that the boundary conditions $u_{out}(0) = 0$ and $u_{out}(\infty) = 1$ are fulfilled, we get the following step response:

$$u_{out}(t) = h(t) \left(1 - e^{-\frac{t}{\tau}}\right) \quad (9)$$

After about 3-time constant RC $u_{out}(t)$ reaches the maximum voltage, or more precisely after 2.3τ 90% of the final amplitude is reached.

Fig 16 shows an analogy for low pass filter. Water level means voltage and water flow current. A capacitor cannot be charged arbitrarily quickly.

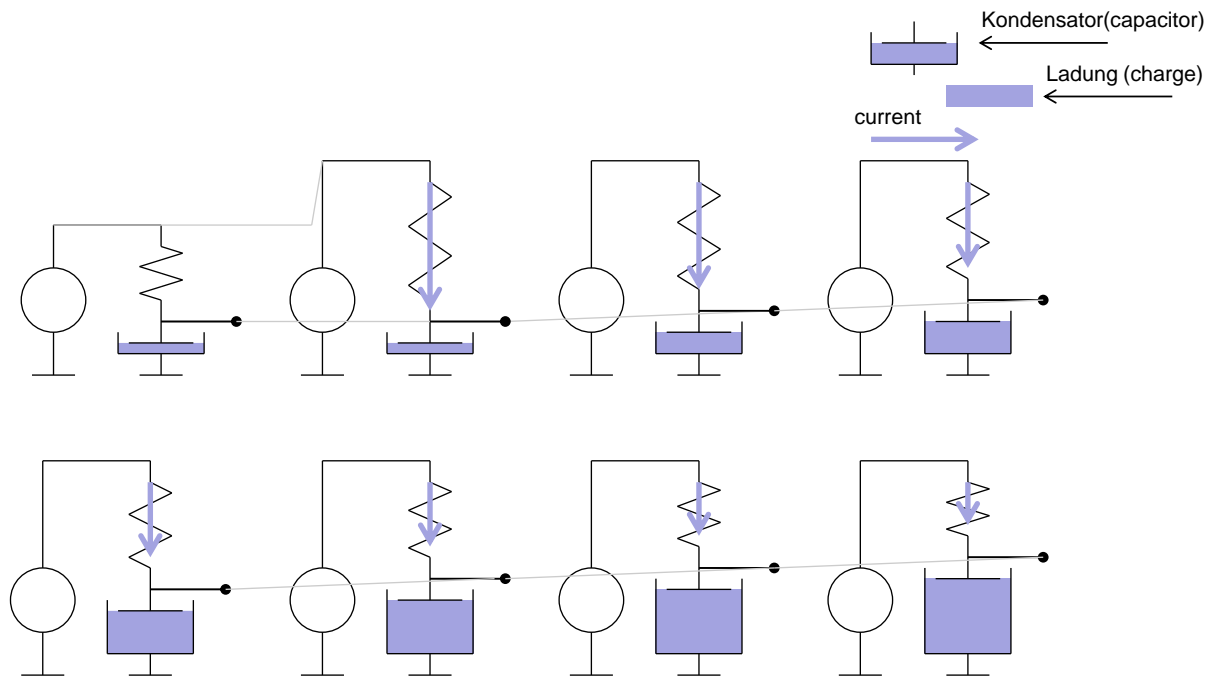


Fig 16: Low pass filter analogy

The sheet shows the pulse time-response of the low pass filter (left) and its Bode plot (right)

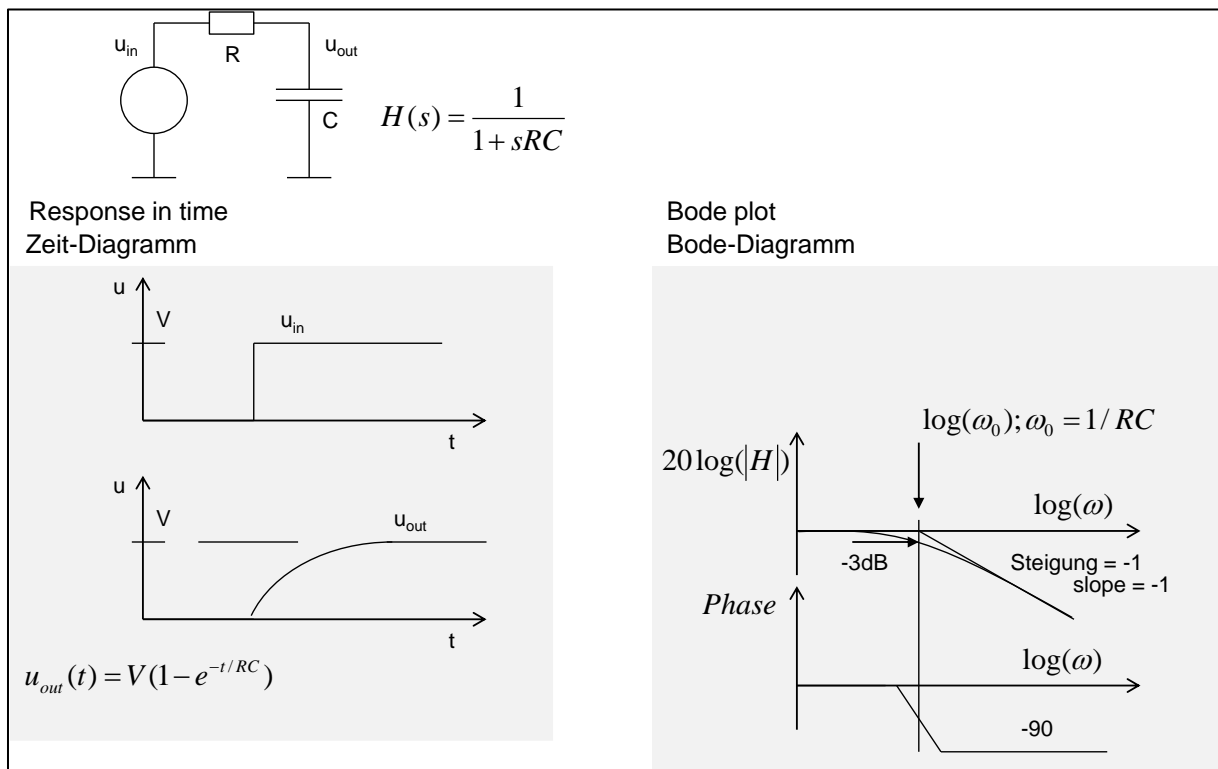


Fig 17: Low pass filter – time dependent response (left) and Bode plot (right)

Bode plot

In a Bode plot (amplitude diagram), $20 \log$ of the absolute value of function $u_{out}(i\omega)/u_{in}(i\omega)$ is plotted versus $\log(\omega)$. In the phase plot, the phase of $u_{out}(i\omega)/u_{in}(i\omega)$ is plotted as a function of $\log(\omega)$.

Bode plots of transfer functions with real zeros and poles are particularly simple. (A pole is the solution of the polynomial in the denominator, a zero is the solution of the polynomial in the numerator.)

In the case of low-pass filter it looks as follows:

For small frequencies the amplitude is 0 dB, i.e. $u_{out} = u_{in}$.

For the frequency = $1/RC$ (called the pole frequency) it holds

$$|u_{out}| = \frac{|u_{in}|}{\sqrt{2}}$$

or $20 \log(|H|) = -3\text{dB}$.

For larger frequencies than pole it holds:

$$|u_{out}| \sim |u_{in}|/(RC \omega)$$

or

$$20\log(|u_{out}/u_{in}|) \sim -20\log(RC) - 20\log(\omega).$$

This corresponds to a line with the -1 slope which cuts the x-axis in the point $1/RC$ (pole frequency).

The solid lines shown in the plot are the asymptotes.

The phase changes from 0 to -90 degrees in the range of $0.1\times$ to $10\times$ pole frequency (within 2 decades).

How to intuitively guess the imstep response without solving of differential equations?

When we examine long time scale, we can remove all the capacitors and short-circuit all inductors. Why? The time derivatives are 0: hence it holds for capacitors $i_c = 0$ and for inductors $u_l = 0$.

We get for the low pass filter $u_{out}(\infty) = 1$.

For short periods of time, the time derivatives multiplied with RC are larger than the terms without derivative (DC-terms). That is why it holds $Z_c \ll R$. Therefore, in the case of a serial connection of R and C , R can be removed and C short-circuited. We get $u_{out}(0) = 0$.

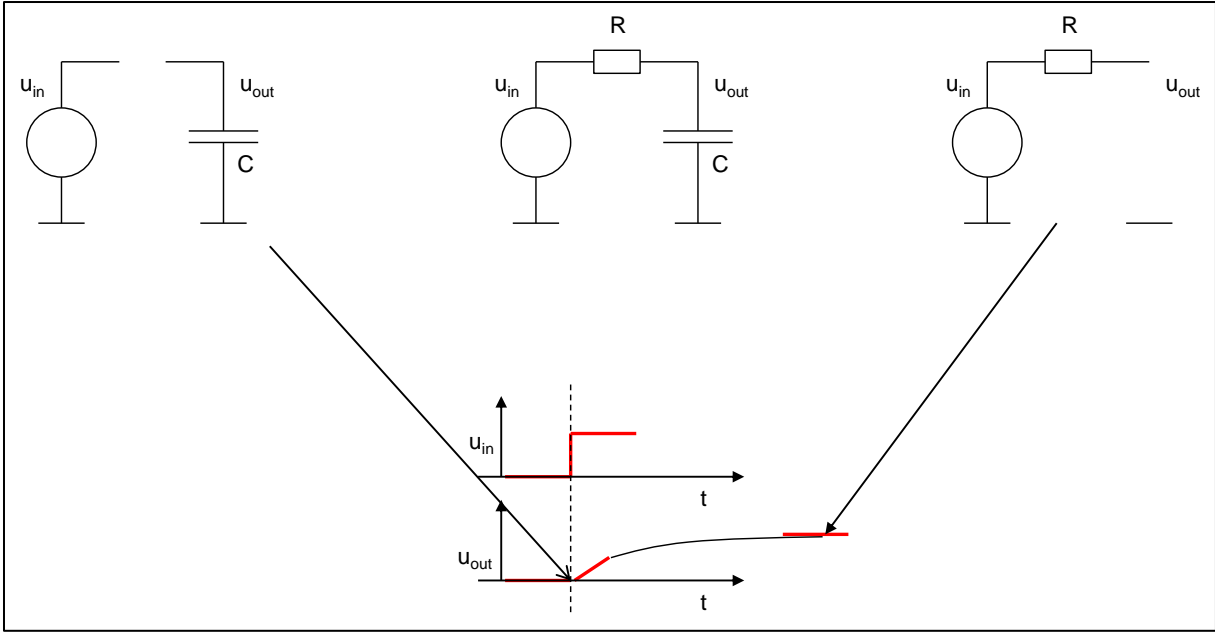


Fig 18: Quick estimation of step response

Fig 19 shows high-pass filter, and its step response and Bode plot.

The transfer function of the high pass filter is

$$u_{out}(s) = u_{in}(s) \frac{sRC}{1+sRC} \quad (10)$$

And the step response

$$u_{out}(t) = h(t)e^{-\frac{t}{\tau}} \quad (11)$$

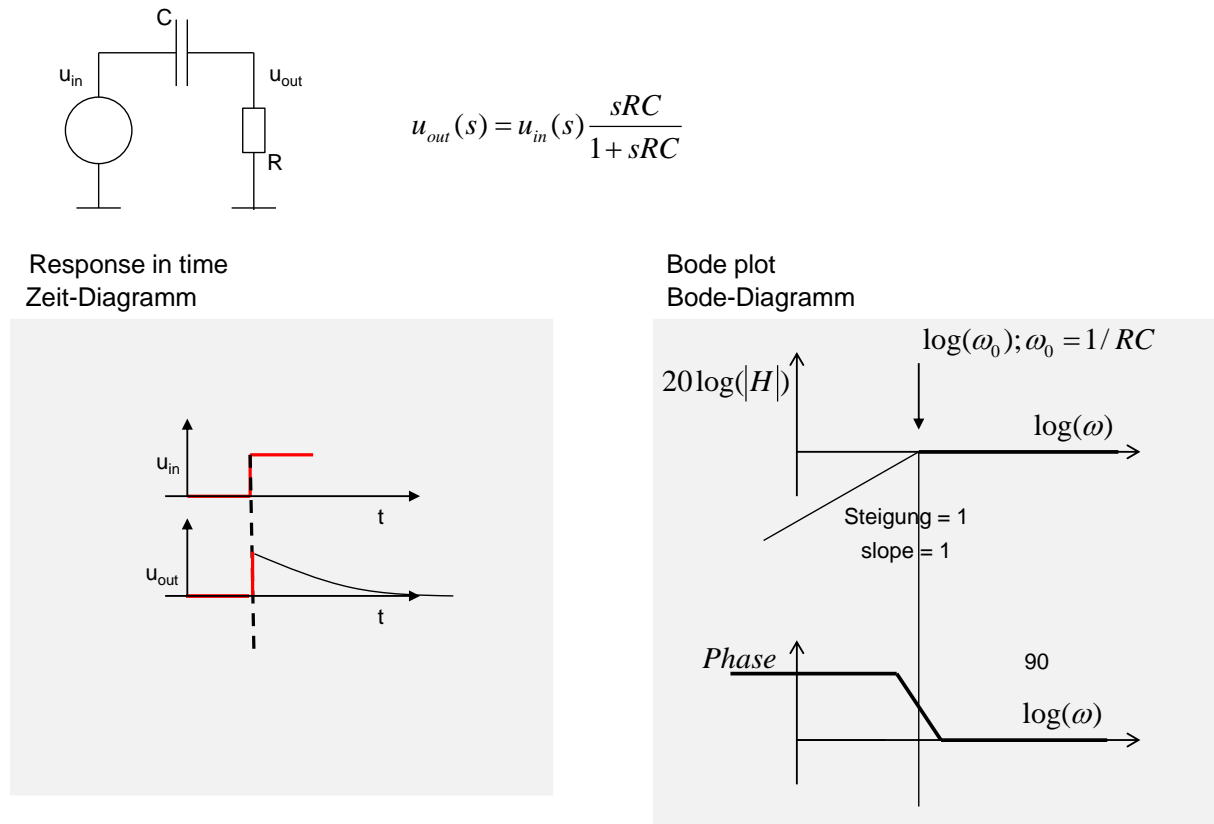


Fig 19: High pass filter – time dependent response (left) and Bode plot (right)

Inverting voltage amplifier as a filter

Voltage amplifier is an important circuit that we also use in exercises with Cadence.

We have analysed this amplifier in the previous lecture. We have neglected the feedback resistor R_{fb} and the resistor R_{out} during the small signal analysis. As a consequence of this neglecting, the formulas we derived in the previous lecture did not depend on frequency. We will now take R_{fb} and R_{out} into account and derive a frequency-dependent voltage gain.

Fig 20 shows the small signal schematic of the inverting voltage amplifier.

C_o is the output capacity that we have, for example, when external circuits are connected to the amplifier.

Input signal is a step function $v_s(t) = h(t)$.

Simplified calculation

Let us now derive the output signal in a simplified way.

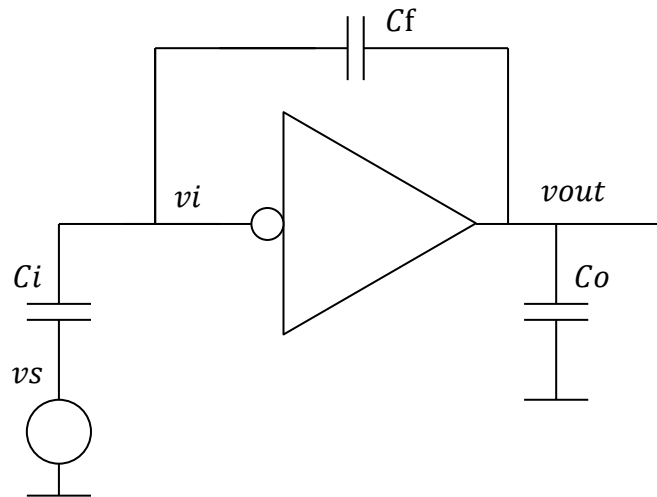


Fig 20: Inverting voltage amplifier

Let us assume that βA_{ol} is for every frequency large (Fig 21). βA_{ol} is in our case frequency dependent.

Case 1: R_f is neglected

Let us also for the moment neglect R_f .

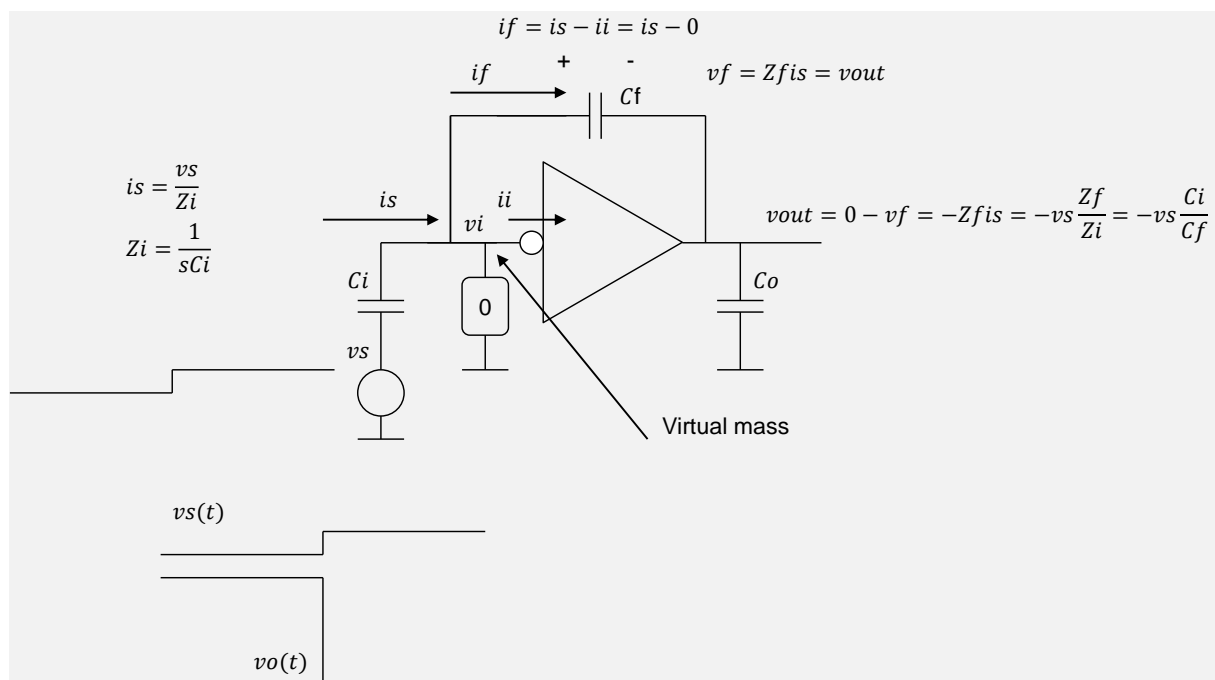


Fig 21: Fast calculation of amplification by assuming $\beta A_{ol} \gg 1$ for every frequency - virtual mass

In this case ($\beta A_{ol} \gg 1$) the point at the input of the amplifier (v_i) is a virtual mass, i.e. the AC voltage in this point is zero.

Let us first calculate the input current $i_s(t)$, as shown in Fig 21.

It holds

$$i_s = v_s / Z_i$$

where Z_i is the impedance of the capacitor C_i . ($Z_i = 1/sC_i$)

It holds also: $i_f = i_s - i_i$.

The current $i_i = 0$, as $v_i = 0$. (v_i is virtual mass)

It follows: $i_f = i_s$.

The voltage across capacitor C_f is:

$$v_f = Z_f \times i_f = Z_f \times i_s.$$

The output voltage is then:

$$v_{out} = v_i - v_f = -v_f = -Z_f i_s = -v_s \frac{Z_f}{Z_i} \quad (12)$$

By substituting the impedances in (12), we get:

$$v_{out} = -v_s \frac{C_i}{C_f} \quad (13)$$

This is the same formula for voltage gain as we derived in lecture 5.

This simplified analysis does not give us the possibility to check whether the condition $\beta A_{ol} \gg 1$ is really fulfilled for higher frequencies.

Case 2: R_f is not neglected

How does the formula (12) change when we take R_f into account?

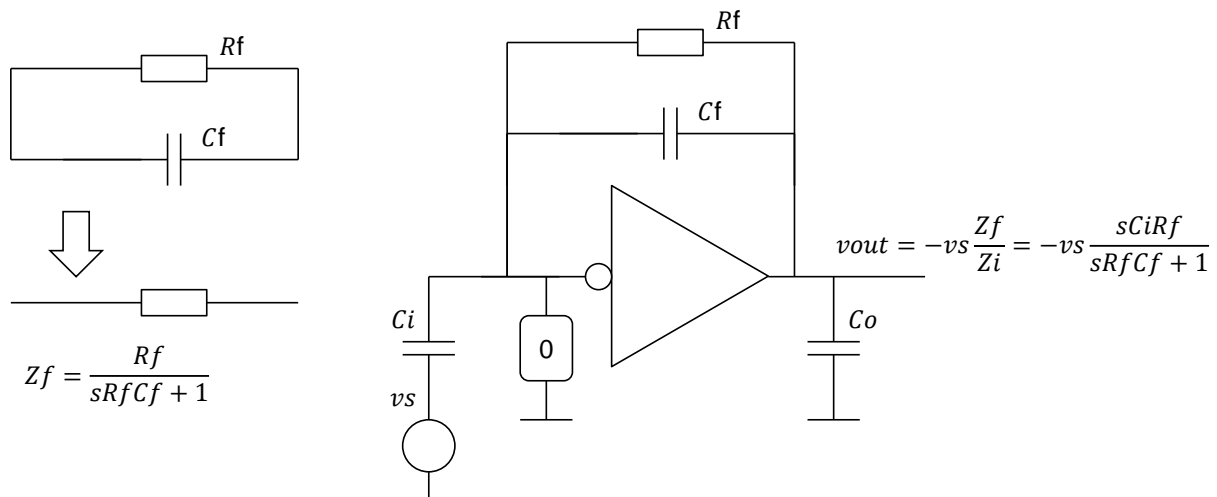


Fig 22: Influence of R_f to the transfer function

The formula (12) is still valid, only Z_f has a different form now (Fig 22):

$$Z_f = \frac{R_f}{1+sR_fC_f} \quad (14)$$

This result can be obtained when we calculate parallel connection of R_f and $Z_{cf} = 1/sC_f$

$$Z_f = \frac{Z_{cf}R_f}{Z_{cf} + R_f}$$

By substituting (14) into (12), we obtain the transfer function:

$$V_{out} = -V_s \frac{Z_f}{Z_i} = -\frac{sR_f C_i}{1+sR_f C_f} \quad (15)$$

How does the response of the circuit to the step function $h(t)$ look like?

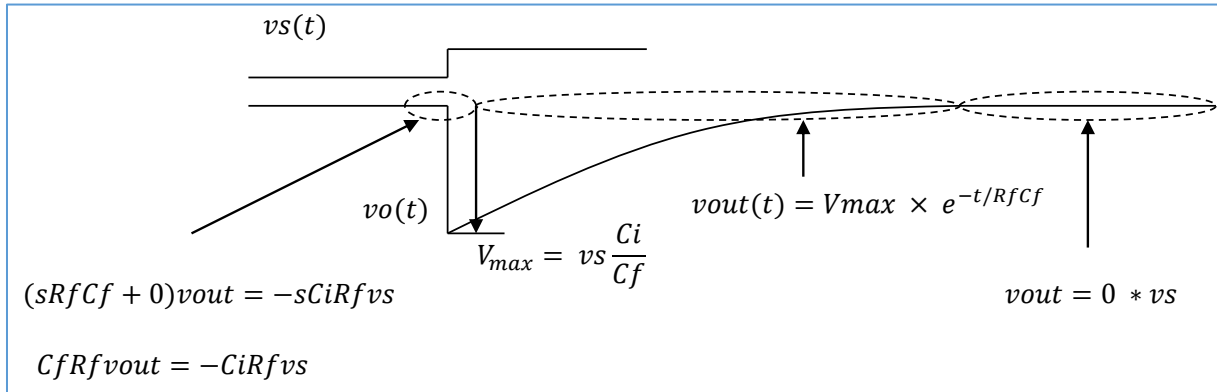


Fig 23: Solutions in different time intervals.

For short time scale, only the terms with s are significant. In this case we get:

$$V_{out} = -\frac{C_i}{C_f}$$

It is the same formula, as (13) that derived without R_f .

For long time scale, we have $s = 0$, and by substituting it in (15) we get $v_{out} = 0$.

In the intermediate times, the characteristic equation has to be solved. The time dependence is

$$v_o = -\frac{C_i}{C_f} e^{-\frac{t}{\tau}} \quad (16)$$

With the time constant $\tau = R_f C_f$.

This is illustrated in Fig 23.

The amplifier behaves as a high pass filter. Compare (15) with (10). Since useful signals often do not have a DC value, an amplifier with the high pass characteristics is well suited. As mentioned before, R_f provides a DC feedback necessary for the production of the work point.

Exact calculation

Let us now perform more accurate calculation where we consider finite amplification $-A$ and the output resistance R_{out} .

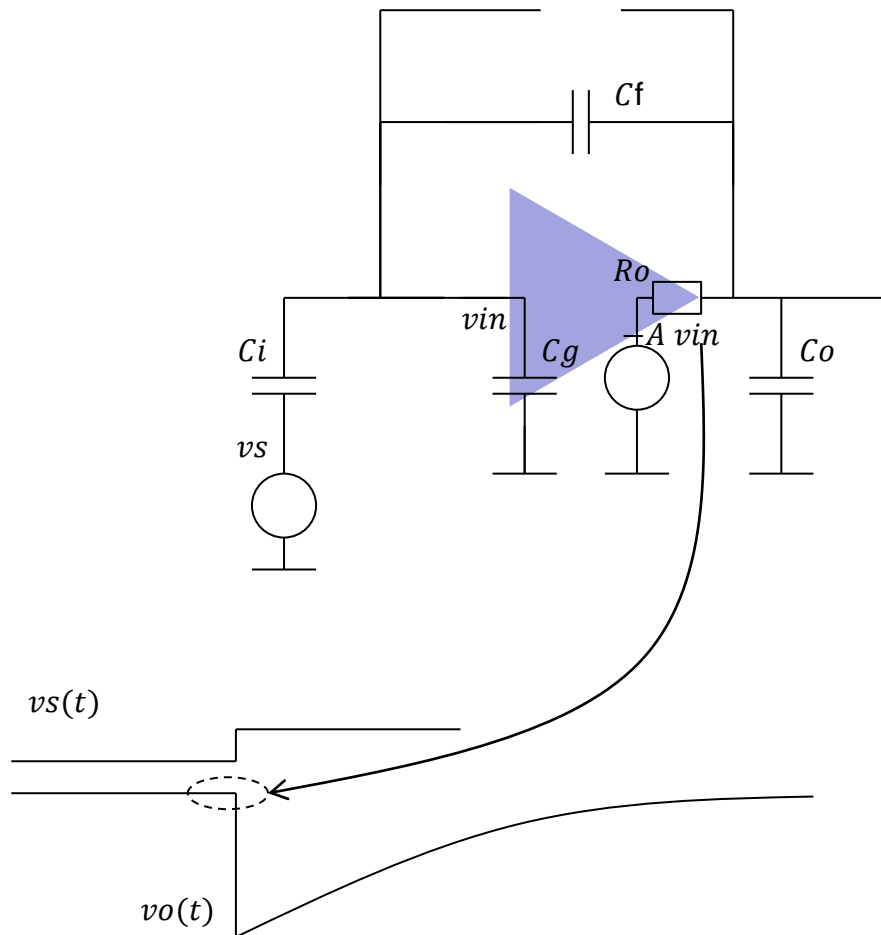


Fig 24: Calculation of step response for short time interval, R_f can be neglected

Case 1: R_f is neglected

We will limit our analysis to short periods, i.e. $t \ll R_f C_f$. R_f does not play a role in this time period and can be neglected (Fig 24). On the other hand, we will not assume that amplification A is infinitely large, but carry out an exact analysis.

Note: a negative amplification $-A$ is an important prerequisite for stability - the feedback must be negative for small frequencies.

We use the feedback analysis, as introduced in lecture 3, by using the Mason's formula (1) and by cutting the feedback path at the amplifier's input.

$$A_{FB} = \frac{FF + A_{in}A_{ol}}{1 - \beta A_{ol}}$$

We will neglect the feed forward factor FF .

On the right side of Fig 25 we show the test circuits for calculating factors A_{in} , A_{ol} and β . The red circle is the test voltage ($=1V$), the arrow shows the place where the voltage is calculated. The value of this voltage is equal to the calculated factor. Usually one voltage needs to be set to zero by shoring to ground, which is marked with red line.

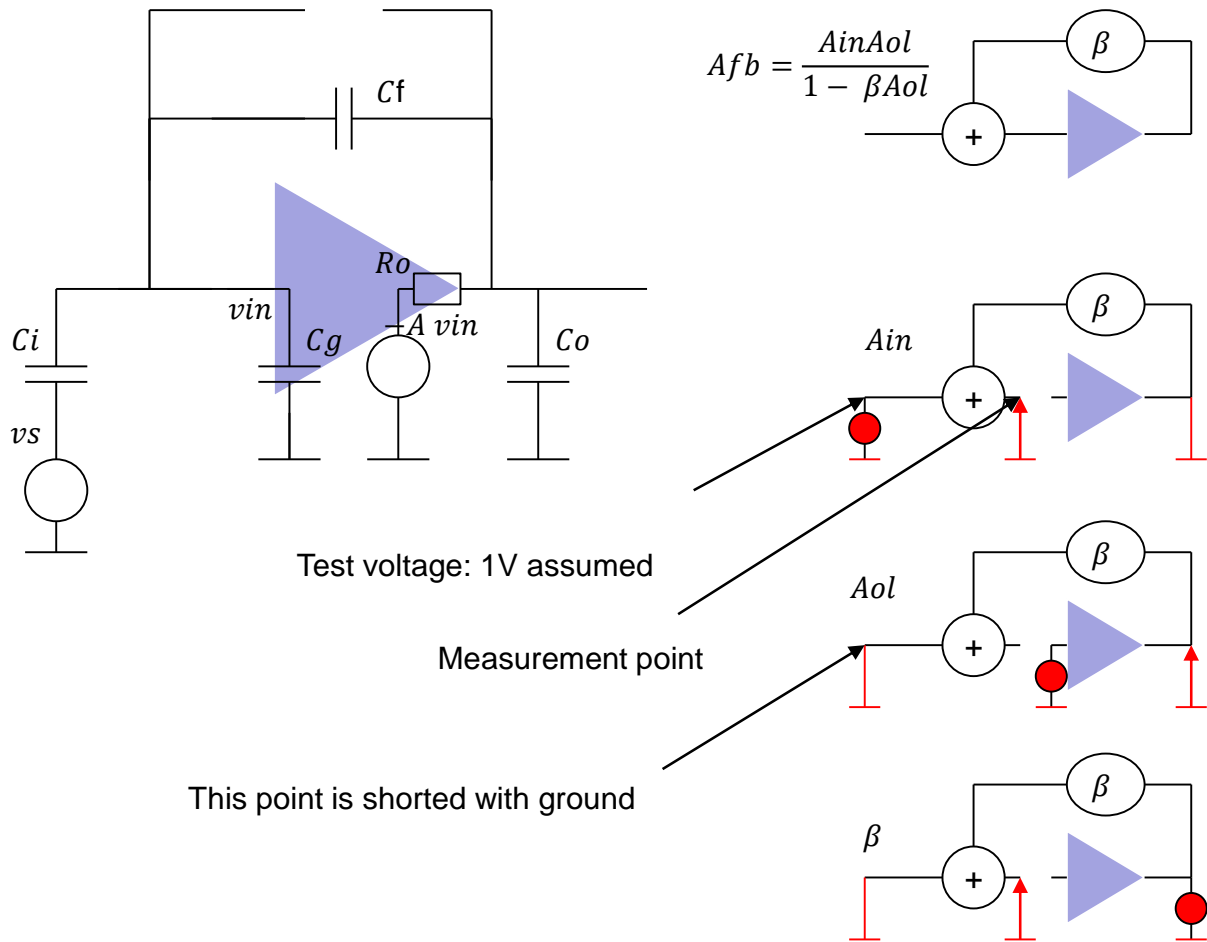


Fig 25: Test circuits for calculation of coefficients in Mason's formula

We will neglect the input capacitance of amplifier, denoted with C_g in Fig 26. The assumption $C_g = 0$ is not always correct and can influence the response. The analysis with finite C_g is presented in the additional pages.

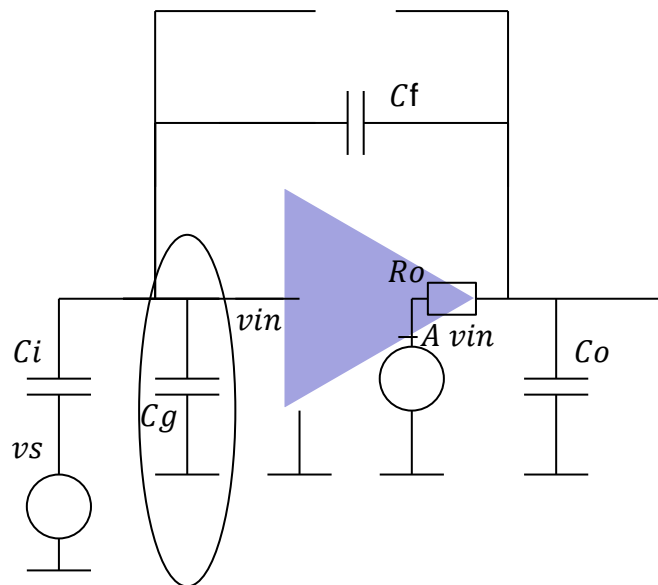


Fig 26: We neglect C_g

In the first step we cut the line between v_{in} and v_{in}^* as shown in Fig 27.

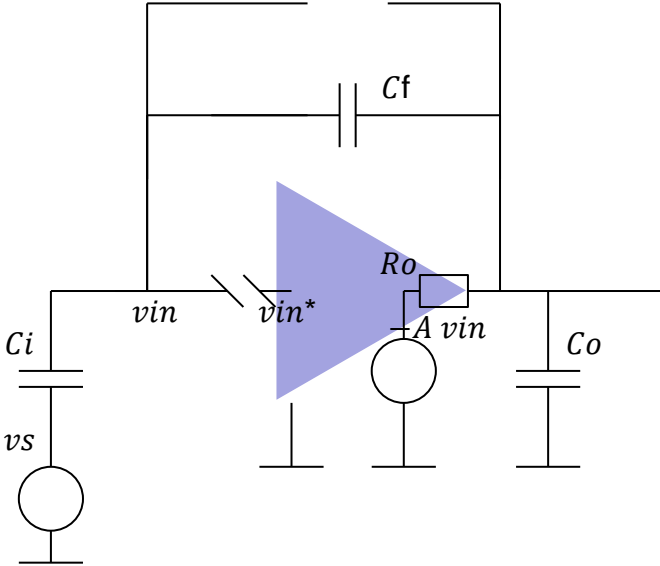


Fig 27: Step 1: feedback path is cut

Let us now calculate the coefficients A_{in} , A_{ol} and β .

Calculation of A_{in}

A_{in} can be derived using formula for voltage dividers, as illustrated in Fig 28. The result is:

$$A_{in} = \frac{sC_i}{s(C_i+C_f)} \quad (17)$$

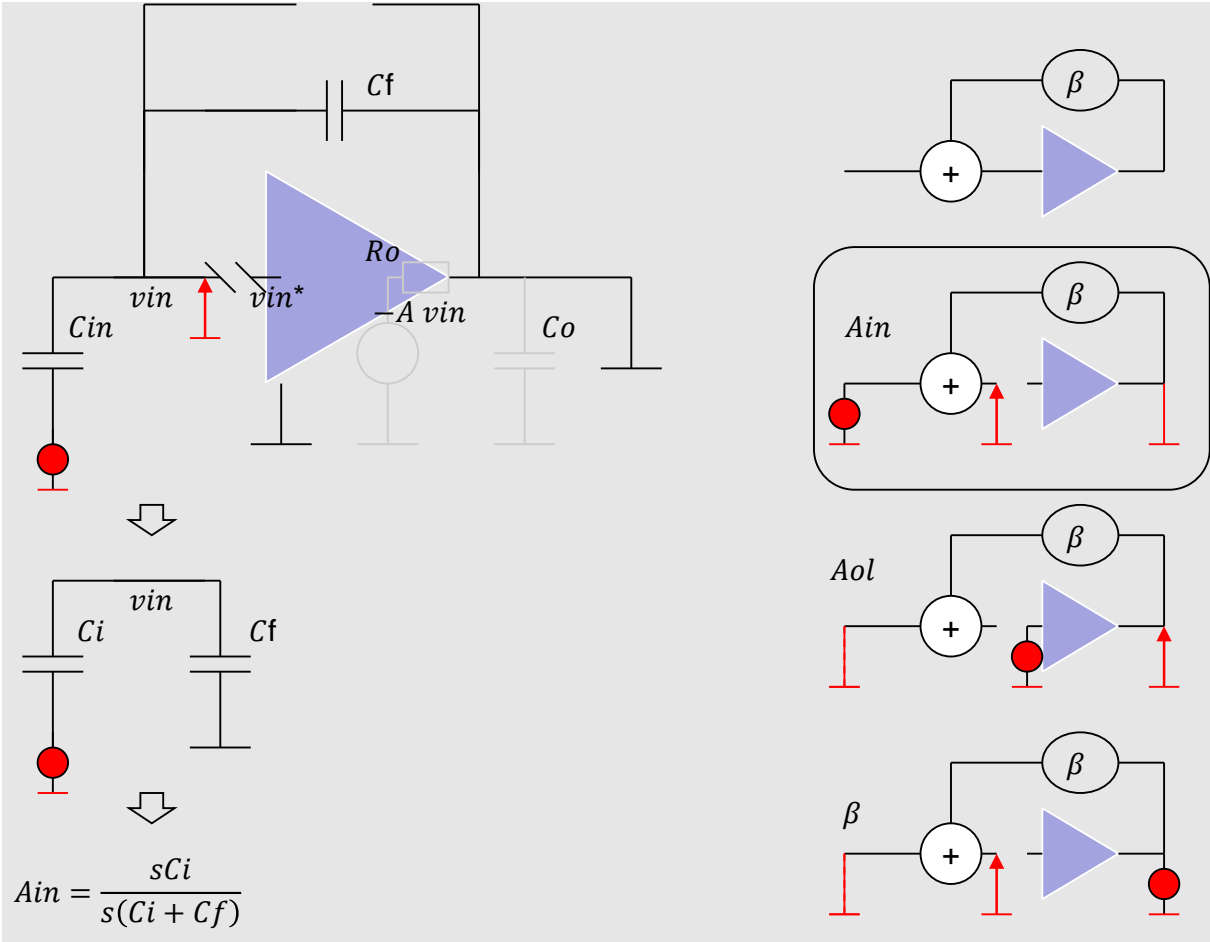


Fig 28: Calculation of A_{in} .

Calculation of A_{ol}

The formula for A_{ol} is a bit longer, Fig 29. The capacitors C_o , C_f and C_i form an equivalent capacitance C_o' , which is attached to R_o . R_o and C_o' form a low pass filter.

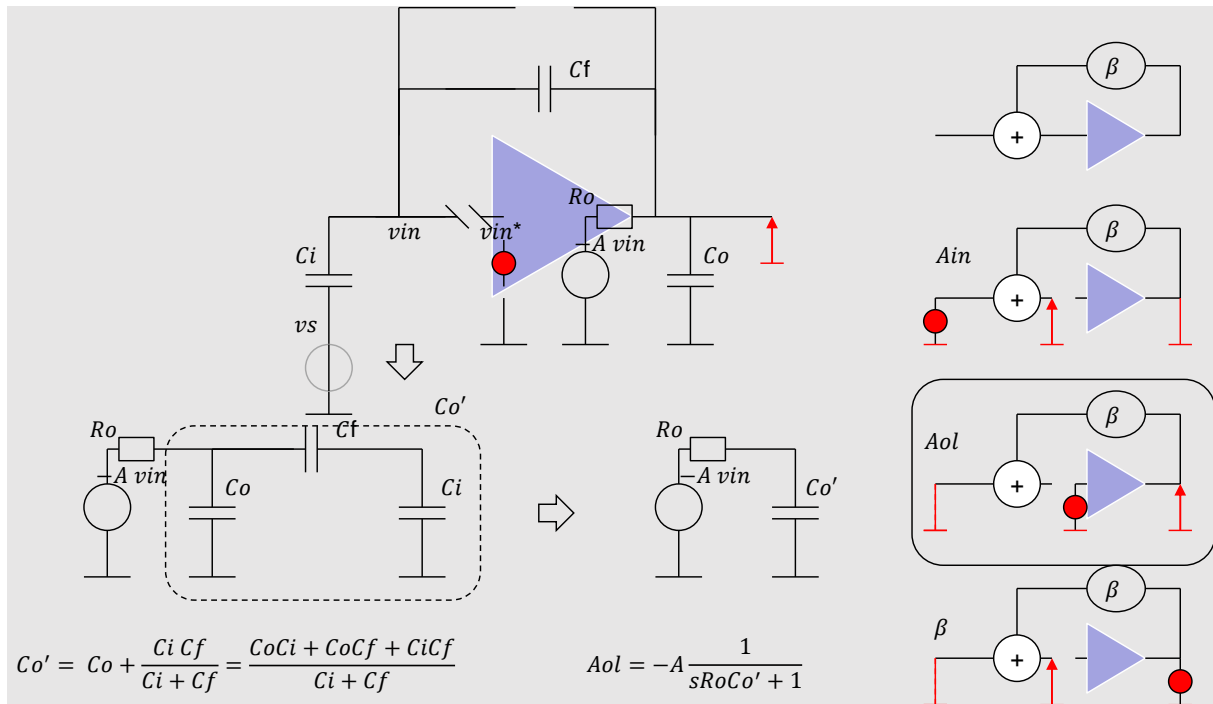


Fig 29: Calculation of A_{ol} .

Therefore:

$$A_{ol} = -A \frac{1}{s R_o C_o' + 1} \quad (18)$$

How big is C_o' ?

It is a parallel circuit of C_o and the series C_f and C_i .

It holds:

$$C_o' = C_o + \frac{C_i C_f}{C_i + C_f} = \frac{C_i C_f + C_i C_o + C_o C_f}{C_i + C_f} = \frac{\sum C_i C_k}{C_i + C_f} \quad (19)$$

$\sum C_i C_k$ is the factor $(C_o C_i + C_o C_f + C_i C_f)$, the sum of all combinations of the capacitances.

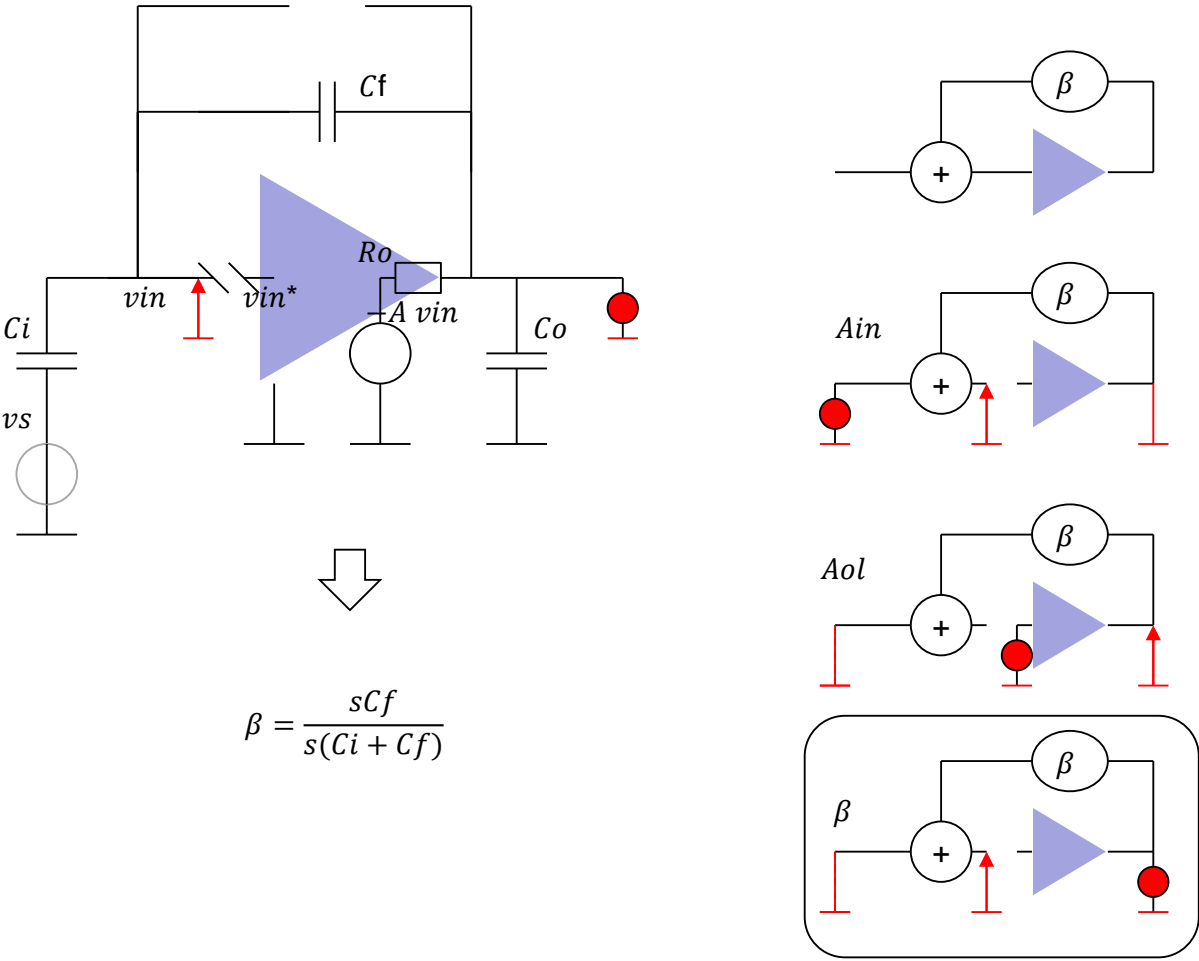


Fig 30: Calculation of beta.

Calculation of beta

Finally we calculate beta (Fig 30). Using formula for voltage divider, we can obtain:

$$\beta = \frac{sC_f}{s(C_i + C_f)} \quad (20)$$

Final result

After calculation of all factors, we can apply the Mason's formula (1). We obtain:

$$A_{FB} = -A \frac{C_i}{C_i + C_f} \frac{1}{1 + \beta A} \frac{1}{1 + \frac{sR_o C_o}{1 + \beta A}} \quad (21)$$

This is shown in Fig 31.

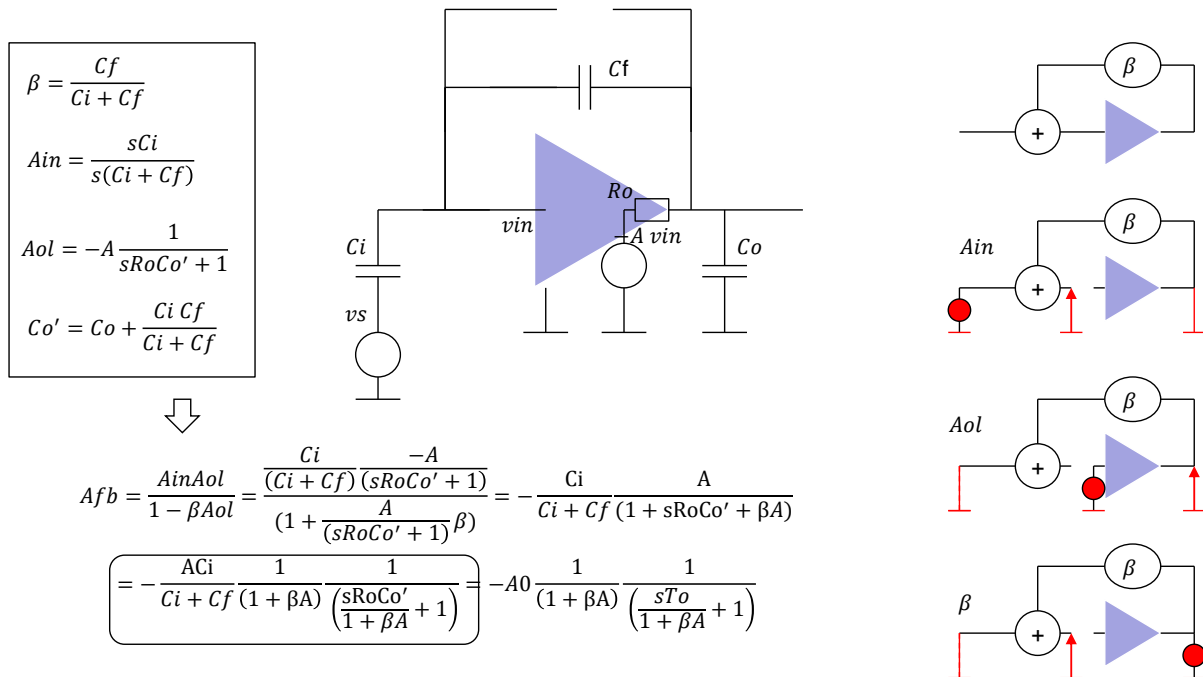


Fig 31: Final calculation with Mason's formula

The formula (21) can be interpreted as follows:

Without feedback we would have gain $A C_i / (C_i + C_f)$ and time constant $R_o C_o'$. With feedback the both are smaller by $1 + \beta A$.

Note that we get + sign in the denominator (i.e. $1 + \dots$) because the amplifier has a negative amplification of $-A$. In other words, in the case of a negative beta A_{ol} , the amplification with feedback is smaller than without. If βA_{ol} were positive for small frequencies (i.e. $\beta A > 1$), the denominator for some frequencies could become 0 (because of the minus sign $1 - \beta A$). In that case, we would have an infinite amplification, or an output signal without an input signal. That means the circuit would oscillate. Therefore, if we do not build oscillators, we should ensure that βA is negative for small frequencies

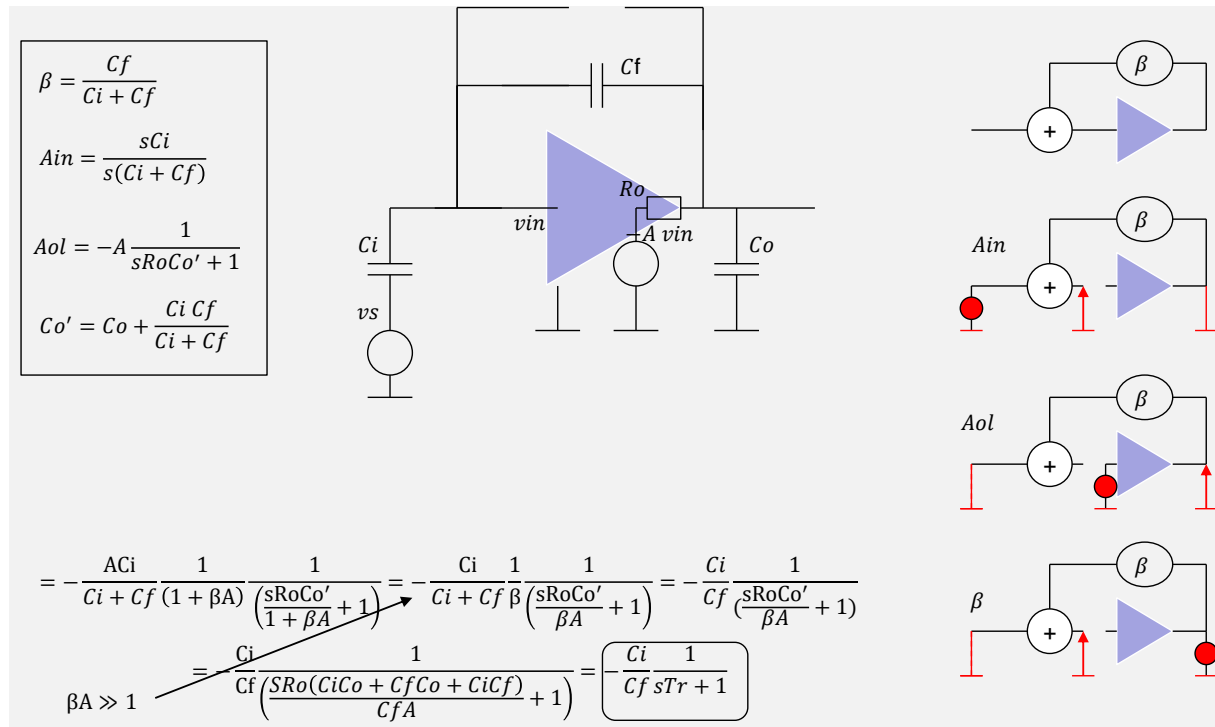


Fig 32: Simplification for the case $\beta A \gg 1$.

Let us now make the assumption $\beta A \gg 1$ (Fig 32).

Note that this is not the same assumption as we made in the simplified analysis, where the assumption was $\beta A_{ol} \gg 1$ for all frequencies. βA is only a part of βA_{ol} .

We obtain from (21):

$$A_{FB} = -\frac{C_i}{C_f} \frac{1}{1 + \frac{sR_o C_o'}{\beta A}} = -\frac{C_i}{C_f} \frac{1}{1 + sT_r} \quad (22)$$

The amplification for low frequencies is $-C_i / C_f$ as we had in lecture 5. We have also the time constant T_r :

$$T_r = \frac{R_o C_o'}{\beta A}$$

When we substitute C_o' :

$$C_o' = \frac{C_i C_f + C_i C_o + C_o C_f}{C_i + C_f}$$

We get

$$T_r = \frac{C_i C_f + C_i C_o + C_o C_f}{A C_f} \quad (23)$$

The formula is easy to remember.

T_r is proportional to the sum of all capacity products ($C_o C_f + C_i C_f + C_i C_o$) and inversely proportional to the amplification A and the feedback capacity C_f .

The transfer function of our amplifier has a low-pass behaviour. Compare (22) with (6).

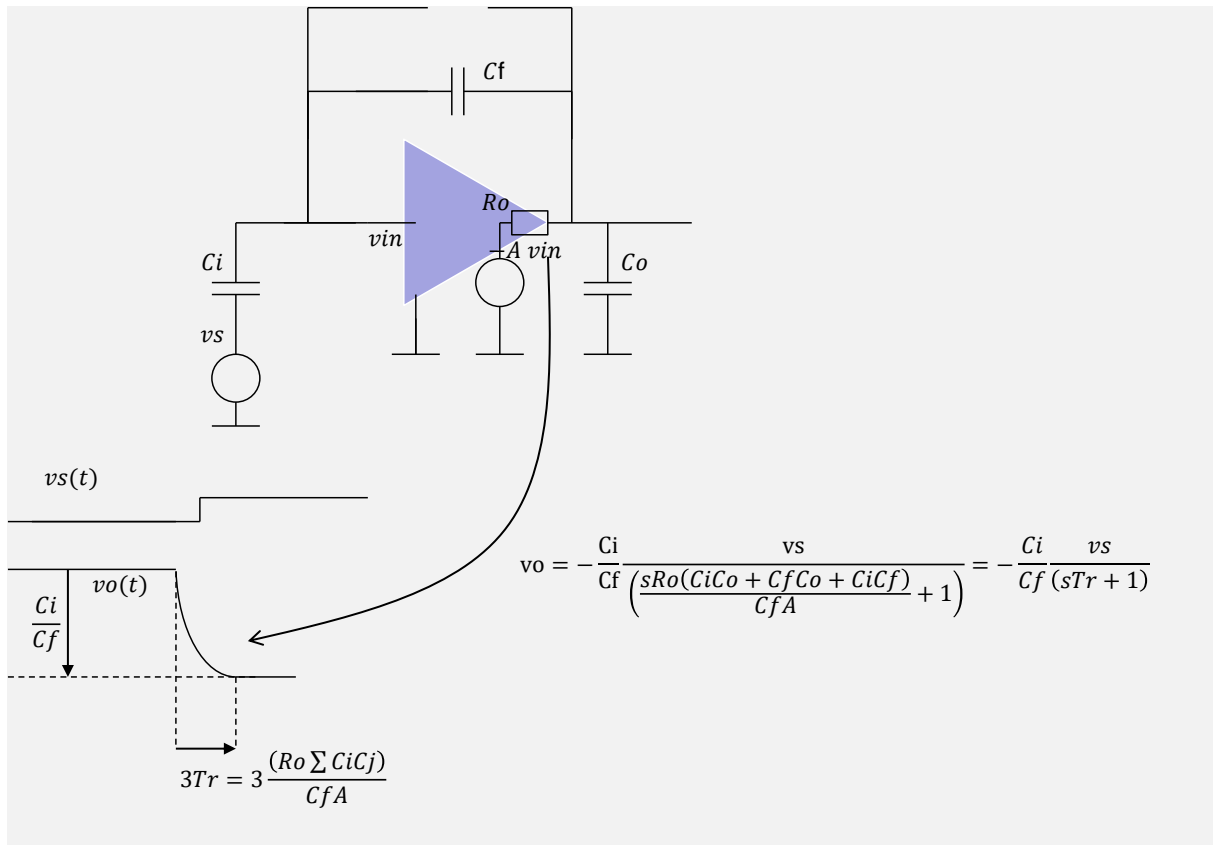


Fig 33: Time constant in the step response

The imstep response can be seen in Fig 33.

The time constant T_r determines the rise time of the output signal. After about $3 \times T_r$ the output signal reaches the amplitude $-C_i/C_f$. If we want our amplifier to be faster, we should choose an amplifier with a larger A .

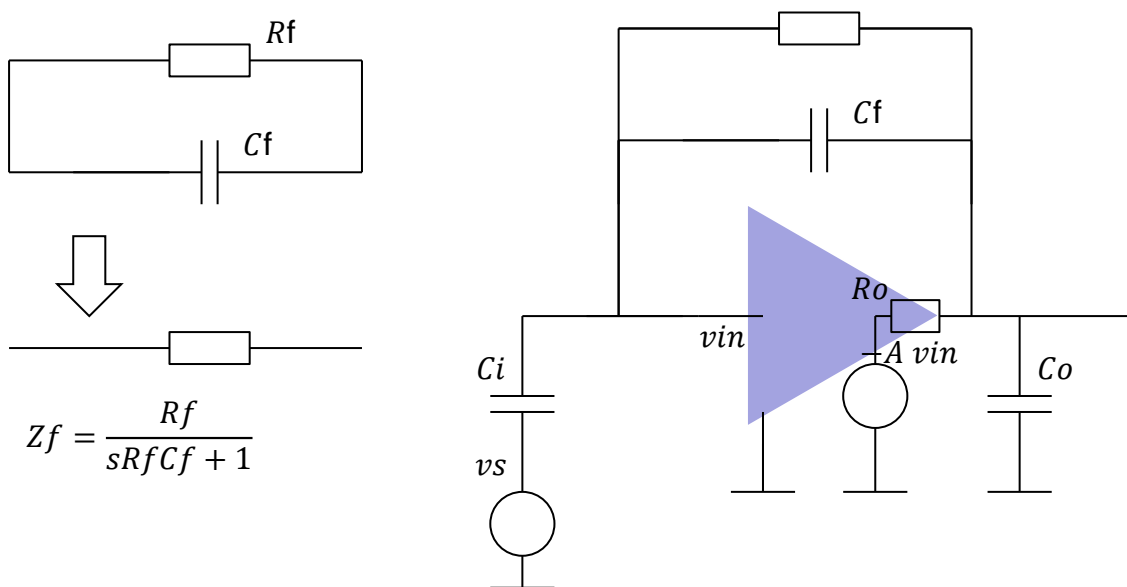


Fig 34: Influence of R_f to step response

Case 2: R_f is not neglected

How does the behaviour of the circuit change if we take R_f into account.

The formula without R_f is (22):

$$v_o = \frac{C_i}{C_f} \frac{1}{1 + sT_r}$$

It can be rewritten as:

$$v_o = \frac{Z_f}{Z_{in}} \frac{1}{1 + sT_r} \quad (24)$$

We can now substitute the equation for impedance of the parallel connection between R_f and C_f

$Z_f = R_f / (sC_f R_f + 1)$ in (24):

$$v_o = - \frac{Z_f}{Z_i} \frac{v_s}{1 + sT_r} = - \frac{sR_f C_i}{1 + sT_f} \frac{v_s}{1 + sT_r} \quad (25)$$

With R_f the amplifier is a combination of high- and low pass filter. It must hold $T_f = R_f C_f \gg T_r$, otherwise the gain of the circuit would be low.

If we draw Bode plot of this transmission function, we can clearly recognise a bandpass behaviour, as shown in Fig 35.

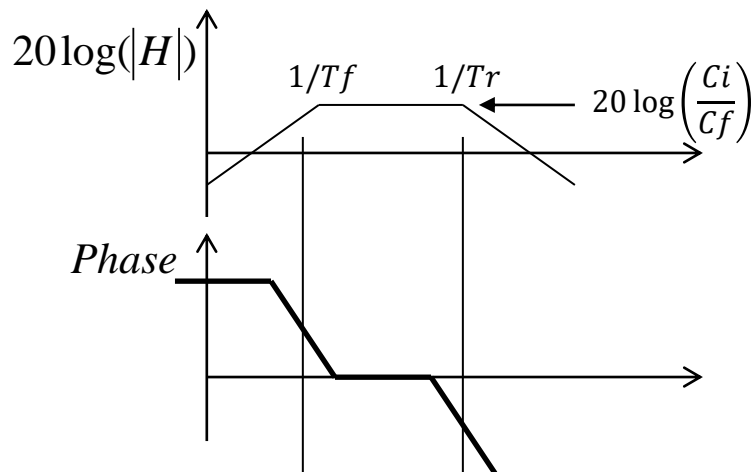


Fig 35: Inverting amplifier with R_f und C_f – frequency response

The step response is shown in Fig 36.

First the output voltage increase almost to the level $-C_i/C_f$, then it drops again at zero. The rise time is approximately $3 \times T_r$ and the fall time $3 \times T_f$.

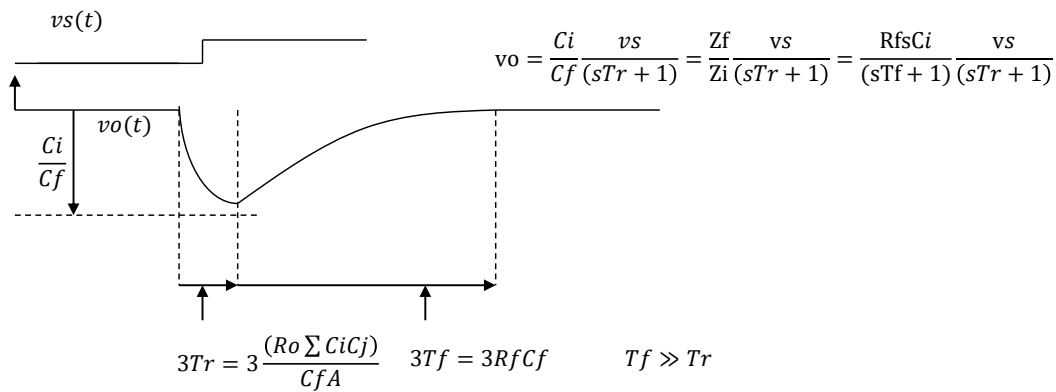


Fig 36: Inverting amplifier with R_f und C_f – step response

Transistor implementation of the amplifier

At the end of this lecture we will discuss the transistor implementation of the amplifier. The amplifier can be realized with a MOSFET and an output resistance.

The small signal model of the amplifier is shown in Fig 37, it is a voltage controlled current source. The amplification is equal to the transconductance (g_m). As mentioned, it is important that a sufficiently large DC bias current flows through the transistor. Only in this case the transistor has a large transconductance.

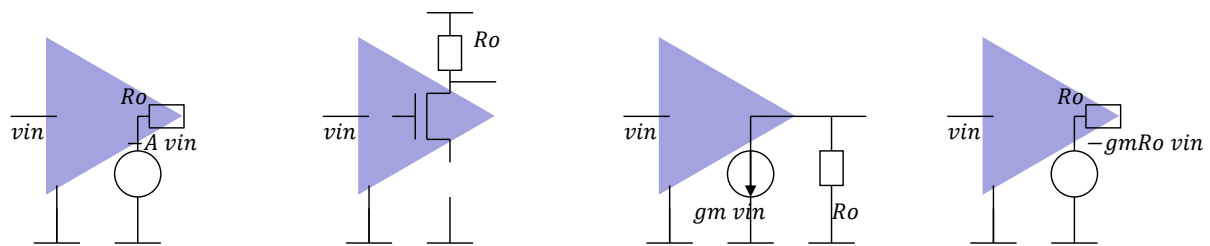


Fig 37: Voltage amplifier implemented with a transistor

We can convert the current source that models transistor into a voltage source, as shown in Fig 37. This is useful as we can then continue to use the formulas we have derived for voltage amplifiers.

The voltage gain can be then expressed as:

$$A = g_m R_o$$

If we replace this in the formula for A_{fb} (22) we get

$$A_{FB} = -\frac{C_i}{C_o} \frac{1}{1+sT_r} \quad (26)$$

with

$$T_r = \frac{C_i C_f + C_i C_o + C_o C_f}{g_m C_f} \quad (27)$$

Interestingly, the time constant T_r does not depend on the R_{out} .

Influence of design parameters to gain and time constant

Let us summarize that how the sizes C_i , C_f , C_o and g_m influence the amplifier in terms of gain and time constant.

When C_i gets larger, the amplification (with feedback) increases but the amplifier gets slower. This follows from (26) and (27) and is illustrated in Fig 38.

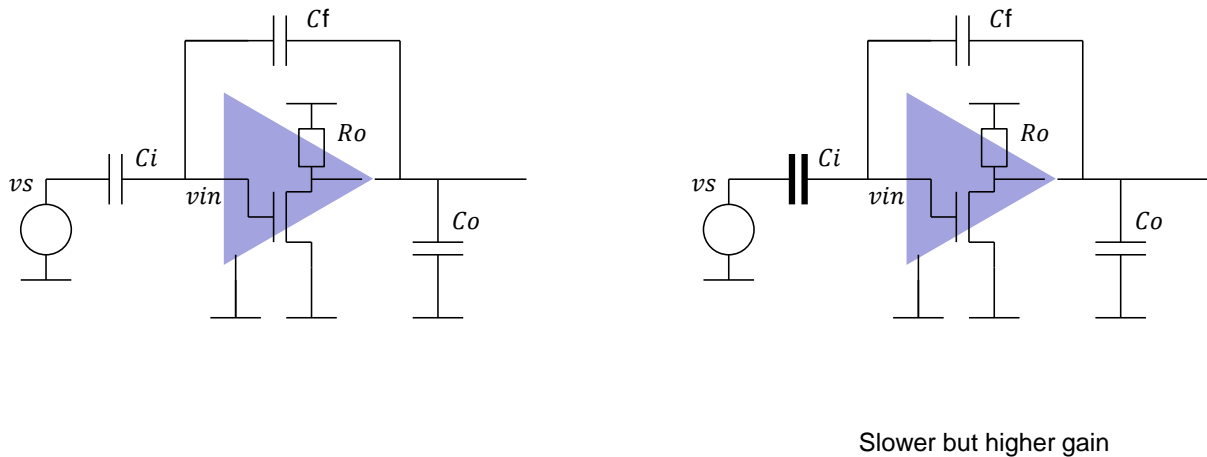


Fig 38: How does C_i influence step response?

If C_f is increased, the amplification with feedback becomes smaller. The amplifier is faster until the time constant reaches the minimum value C_o/g_m . This is shown in Fig 39.

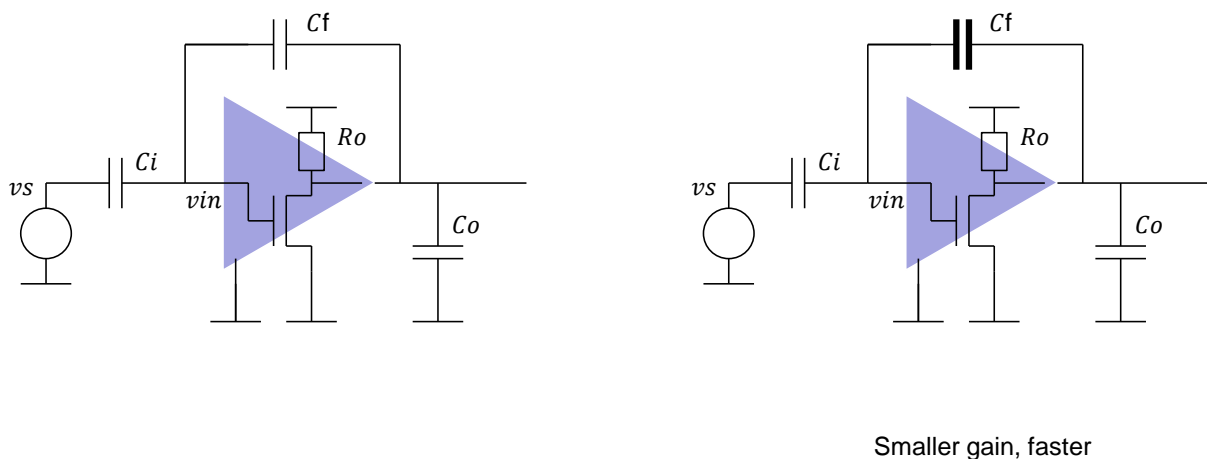


Fig 39: How does C_f influence step response?

Larger load capacity C_o makes the amplifier slower, as shown in Fig 40.

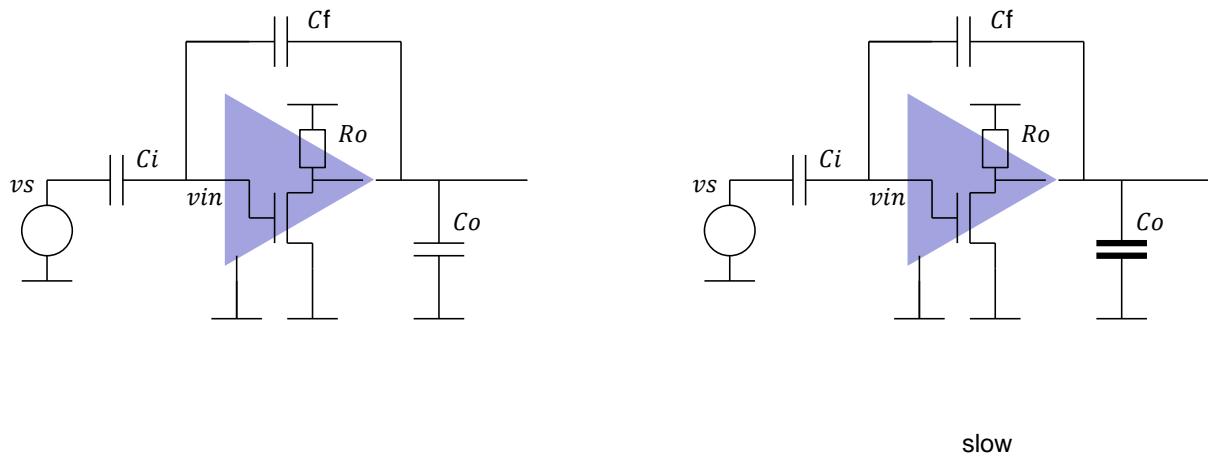


Fig 40: How does C_o influence step response?

What can we do about it? We can simply connect several amplifiers in parallel (Fig 41). Then the equivalent g_m gets larger and the time constant T_r gets smaller, as can be seen from (26) and (27). Note that it is not sufficient to connect several transistors in parallel as they would then have smaller bias current and the equivalent g_m would not be a factor n greater. It is therefore important to increase both the transistor size and the bias current, which is achieved by the use of parallel connection.

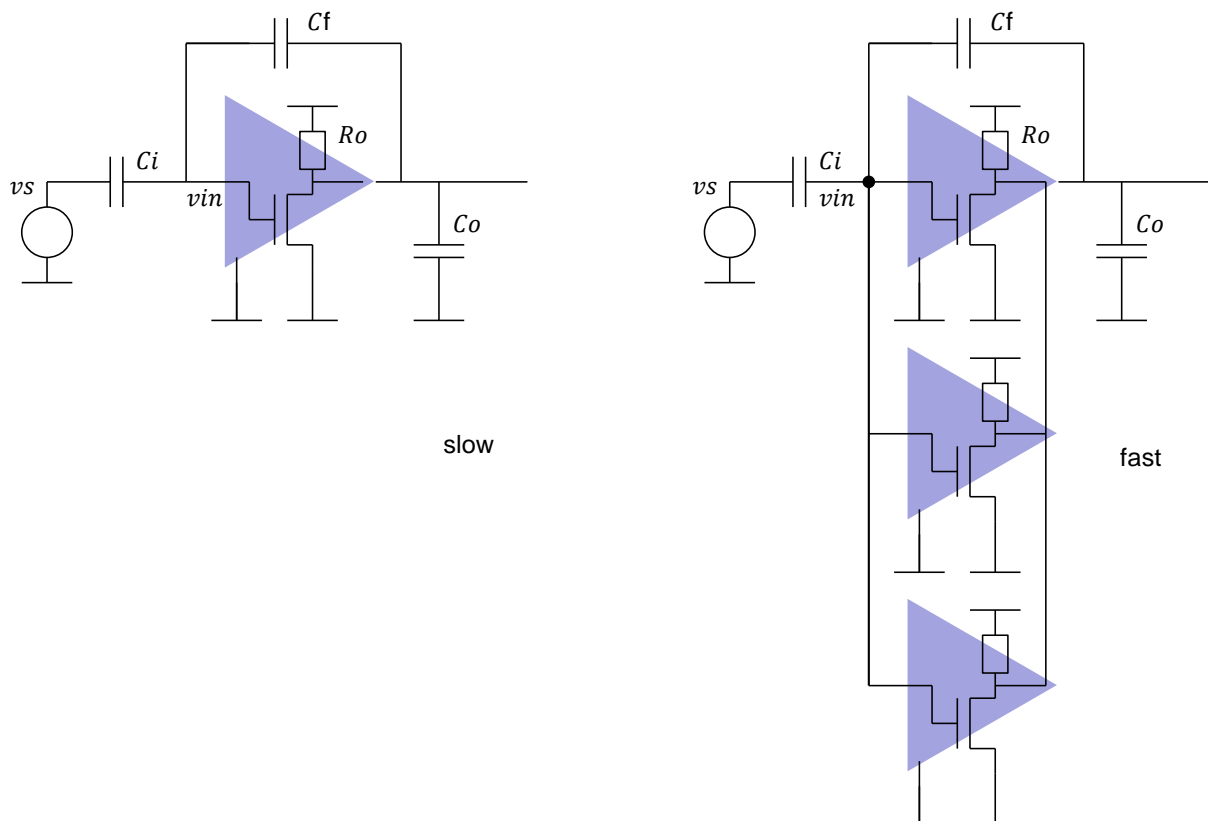


Fig 41: Use of parallel connected amplifiers make the step response faster

Additional pages

Transfer function of inverting amplifier with input capacitance C_g

We have neglected in the previous analysis the gate source capacitance of the input transistor. Fig 42 and Fig 43 show the formulas with C_g . βA und A_{in} contain both the capacitance $C_i' = C_i + C_g$ in numerator, instead of C_i . Both factors are equally diminished. For $\beta A \gg 1$ we get the same gain as for $C_g = 0$. The only difference is the time constant T_r . It is now larger, since we have a large capacitance C_i' instead of C_i .

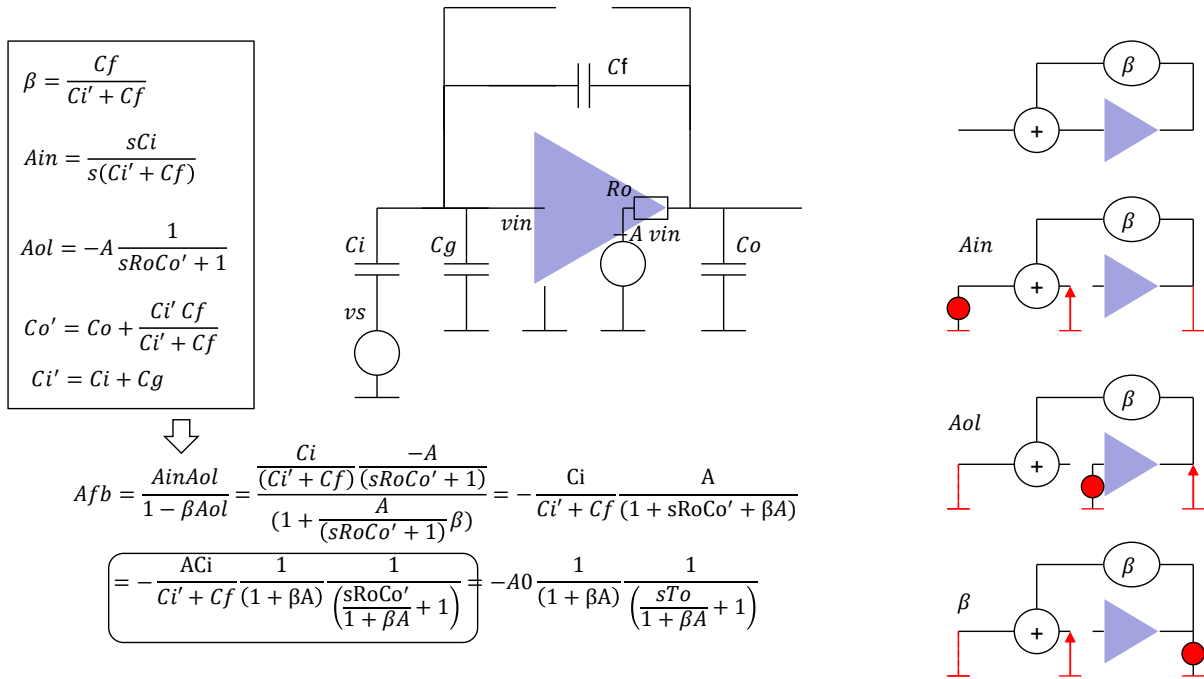


Fig 42: Calculation of step response when C_g is taken into account

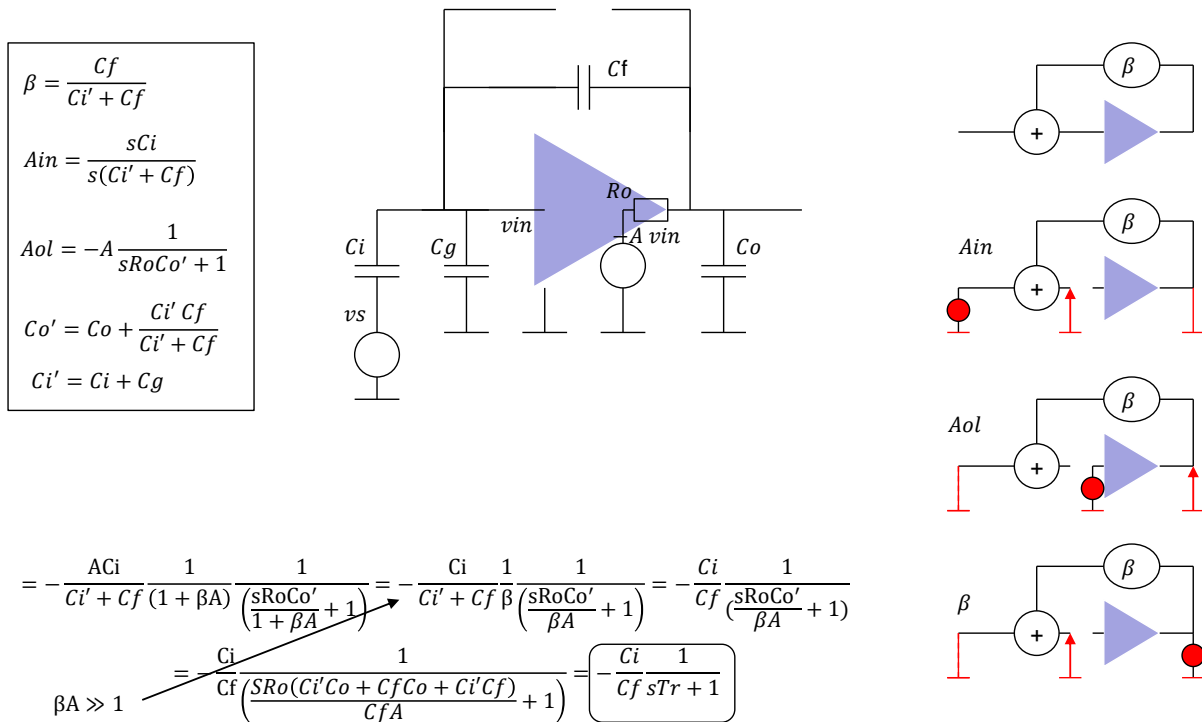


Fig 43: Calculation of step response when C_g is taken into account, case $\beta A \gg 1$

Method for fast estimation of coefficients in characteristic polynomial

A circuit with n independent capacitors has a polynomial of nth order in the denominator

Independent capacitors are the capacitors that can have arbitrary initial voltage.

Polynomial in numerator has a smaller or equal order than the polynomial in denominator.

The polynomial in denominator determines the response to initial conditions (relaxation of the circuit), i.e. how the capacitors get discharged after the circuit is “left alone”. The polynomial in numerator determines the response to the input signal or to switching on of the signal.

The coefficient a1 can be calculated using the following formula:

$$a_1 = \sum C_i R_i^0$$

R_i^0 is the resistance seen by C_i (Fig 44)

The coefficient a2 can be calculated in the following way

$$a_2 = \sum C_i C_j R_i^0 R_j^i$$

R_j^i is the resistance seen by C_j when C_i is shorted (Fig 45).

Although the characteristic polynomial has n coefficients, only a1 and a2 coefficient are important. They are determining two time constants: the dominant time constant $\tau_1 = a_1$ and the second time constant: $\tau_2 = a_2/a_1$. The dominant time constant determines how fast is the step response and how large is the system bandwidth. The ratio between a2 and a1 determines whether a system is stable if a feedback is introduced. Coefficient a1 must be larger than a2/a1.

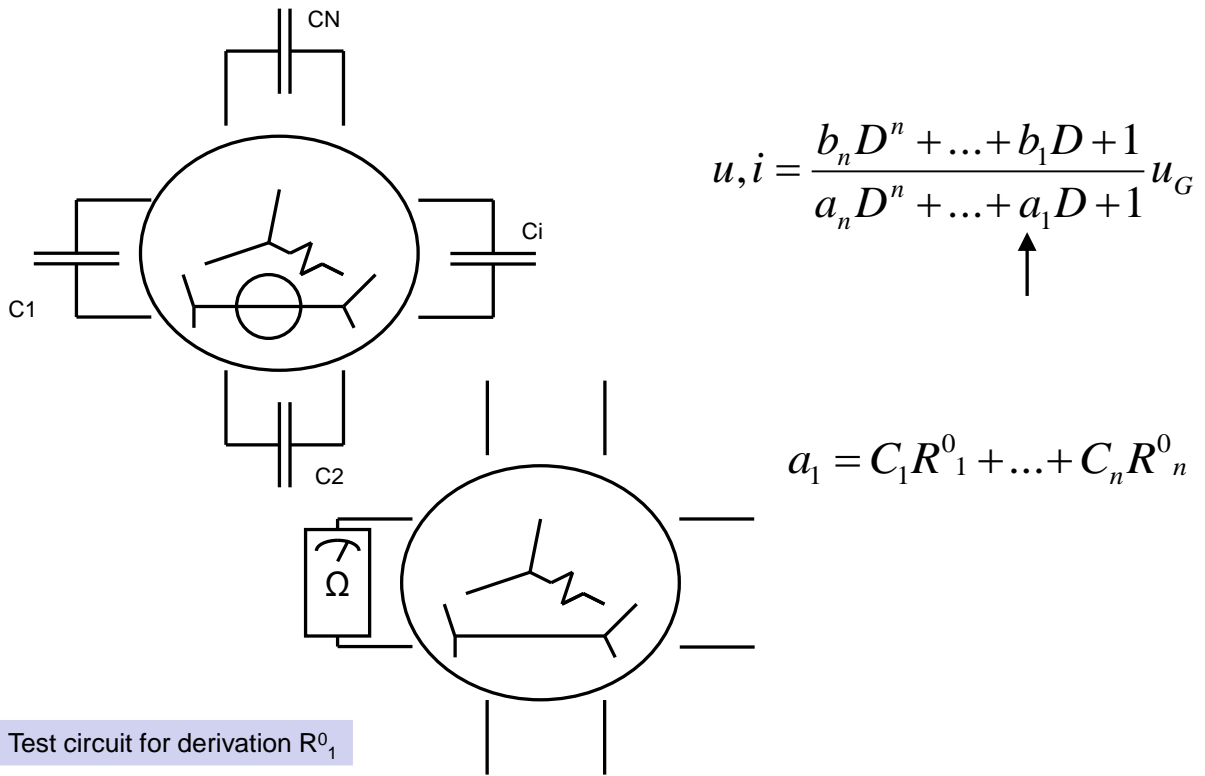


Fig 44: Formula for fast calculation of first coefficient in the characteristic polynomial

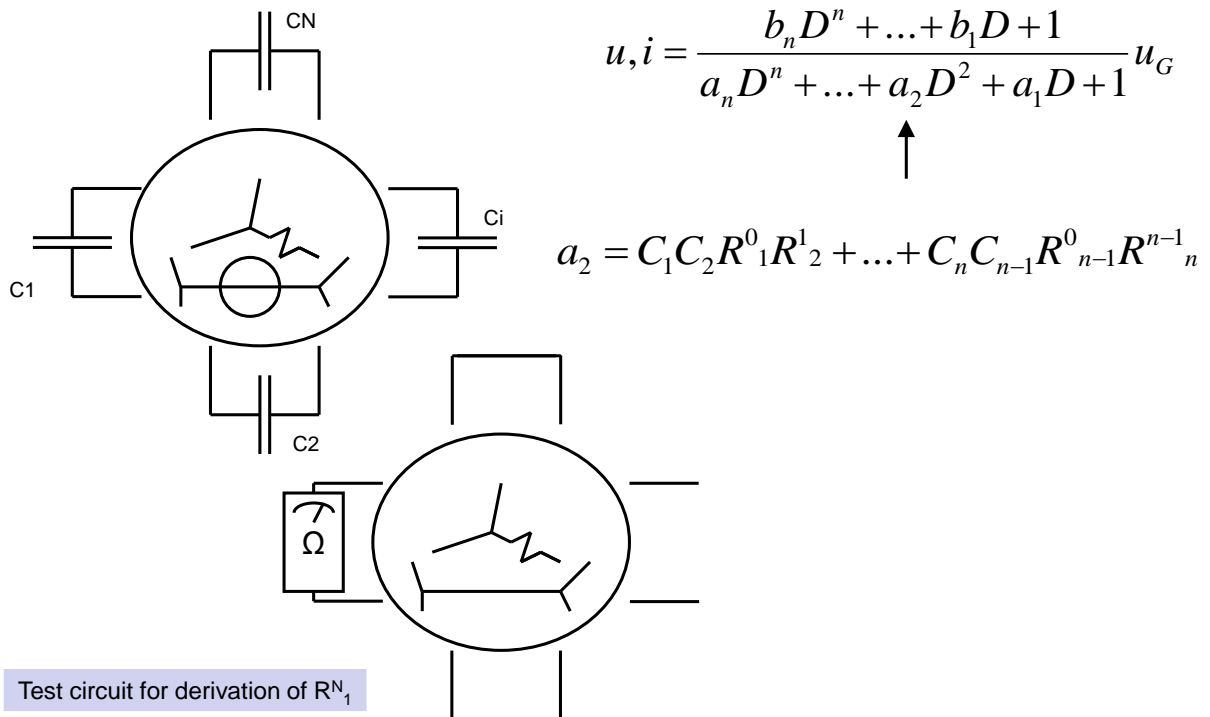


Fig 45: Formula for fast calculation of second coefficient in the characteristic polynomial