Lecture 5

The themes of the lecture are

AC and DC analysis

Inverting Amplifier with feedback

Feedback analysis with Mason's gain formula

AC and DC analysis

The voltages and currents in electronic circuits often have an average value (DC-value, DC-part of the voltage/current) that is different than zero. This DC part is needed to ensure that the transistors are in the correct working (operating) condition. The signal part (the information) of a voltage/current has in most cases smaller amplitude, we call it the AC part (the small signal). The AC part has a time dependence.
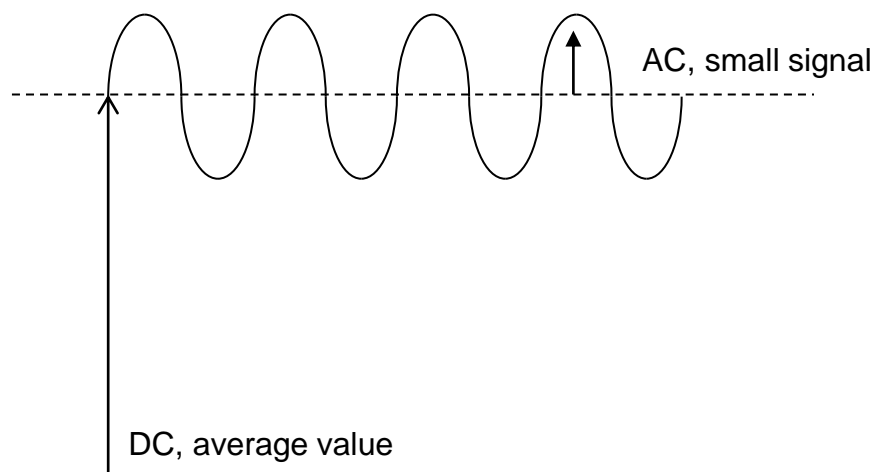

Fig 1: AC and DC

The following notation is used:

$$v_{OUT}(t) = V_{out} + v_{out}(t)$$

$v_{OUT}(t)$ is the actual voltage

V_{out} is the DC part

$v_{out}(t)$ is the small signal (AC part)

If the AC part has a significantly smaller amplitude than the DC part, a small signal analysis is useful. The circuits are first analysed statically for DC voltages (and DC currents). After that, AC- (or small signal-) models are derived (the characteristics of the components are linearized) and finally the AC voltages and AC currents are calculated using these linear AC models.

Note that only AC part of voltage/current contains information, the DC part is only there to ensure proper functionality of the transistors.

An amplifier should ideally not amplify the DC part of a voltage (its average value), it should just amplify the AC part, because, as mentioned, only this part contains information, Fig 2.
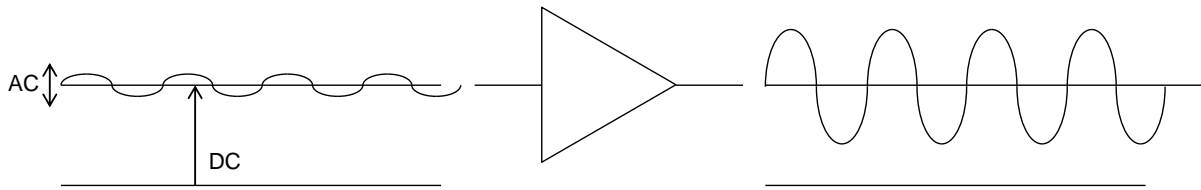
Fig 2: Amplifier amplifies only AC part

An amplifier changes the signal form because its amplification depends on frequency, as shown in Fig 3.
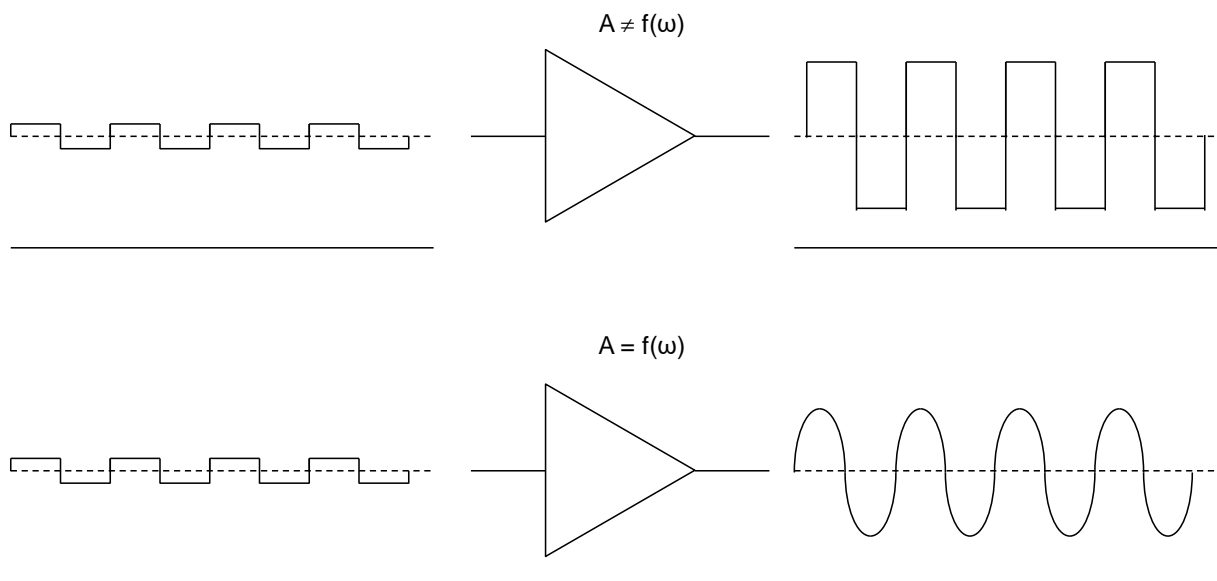
Fig 3: The amplification with and without frequency dependence

Small and large signal models

Electronic circuits are almost always nonlinear; the characteristics are nonlinear and also the capacitances.

It is difficult to solve nonlinear differential equations.

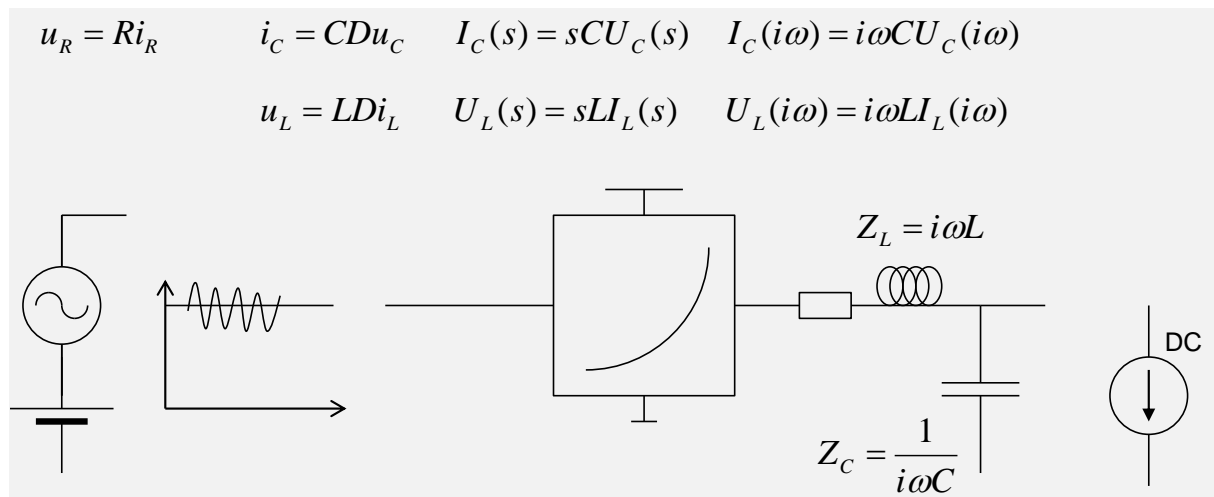Therefore, the analysis of circuits is divided into DC and AC analysis.

$$u_R = Ri_R \qquad i_C = CDu_C \qquad I_C(s) = sCU_C(s) \qquad I_C(i\omega) = i\omega CU_C(i\omega)$$

$$u_L = LDi_L \qquad U_L(s) = sLI_L(s) \qquad U_L(i\omega) = i\omega LI_L(i\omega)$$

$$Z_L = i\omega L$$

$$Z_C = \frac{1}{i\omega C}$$

Fig 4: Nonlinear circuits, R, L and C parts equations, impedances

Step 1 DC analysis

We first calculate the DC voltages in a circuit. For this we use the nonlinear models of components, without capacitances and inductances, as shown in Fig 5. (C is removed, L is short circuited)
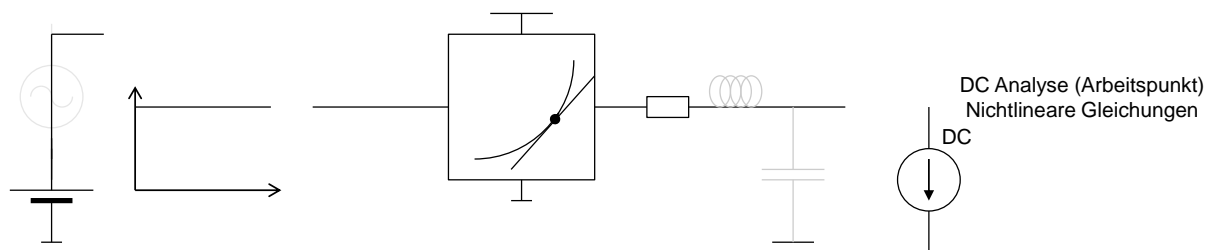
DC Analyse (Arbeitspunkt)
Nichtlineare Gleichungen

Fig 5: DC analysis, working point

Step 2 AC analysis

Then the characteristics and capacitances/inductances are linearized in the vicinity of DC operating points. This is how the small signal models are derived.

We ignore all DC voltages and currents and calculate only the small signals. In a small signal circuit, all DC voltage sources are switched off and all permanent current sources are removed, as showed in Fig 6.
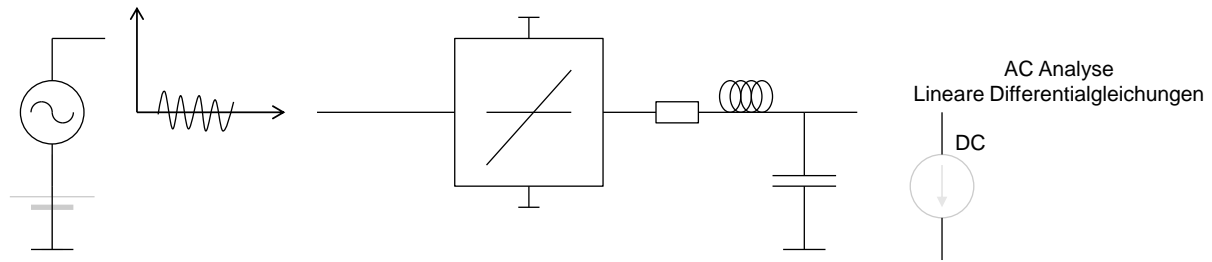
Fig 6: AC analysis

This separation on AC- and DC analysis is not always necessary. Simulation software tools can directly solve nonlinear differential equations. AC and DC analysis is however useful if we want to understand the behaviour of a circuit and for circuit design

Let us start with a basic circuit, the amplifier.

Amplifier

In its simplest version, the amplifier has an input and an output (single-ended amplifier). The negative supply voltage is connected to ground (GND), the positive supply voltage we call VDD, Fig 7.
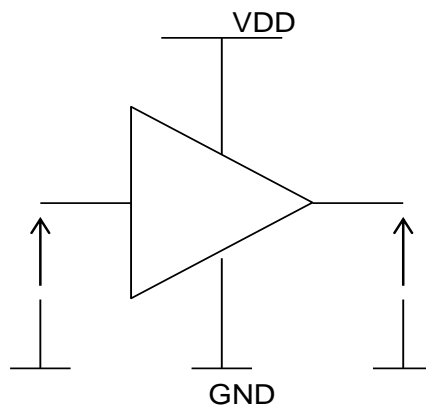

Fig 7: Single ended amplifier

We can characterize an amplifier if we connect a voltage source at the input and measure the output voltage with an ideal voltmeter

Fig 8 shows a typical input-output characteristics of an amplifier. The amplification (voltage gain) is only in a small region of the characteristics high (and negative). Let us call it the active region. For higher or lower input voltages the amplification is low - the amplifier saturates.
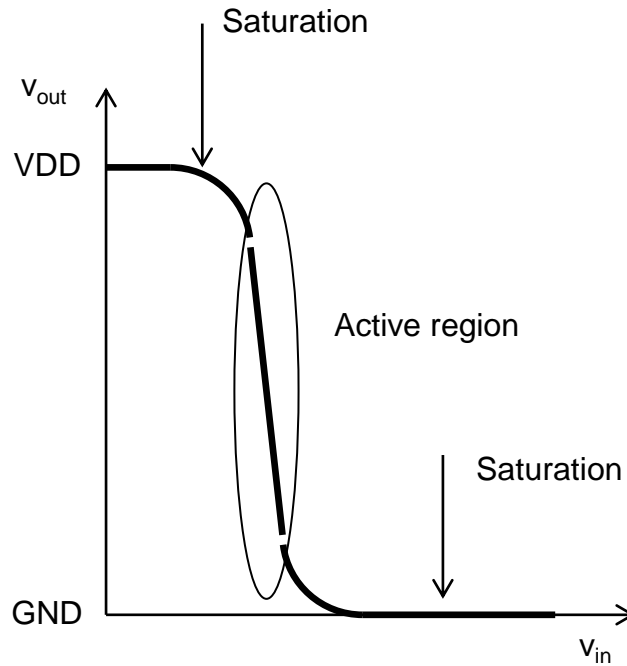
Fig 8: Input output characteristics of the amplifier

We can realize an amplifier if we connect a resistor R_{load} to a transistor as shown in Fig 9.

The resistance has two tasks: firstly to bias the transistor and to create a proper DC operating point, secondly to convert the output current into a voltage.

We can derive the input output characteristic mathematically (with equations) or graphically (using of graphical characteristics).
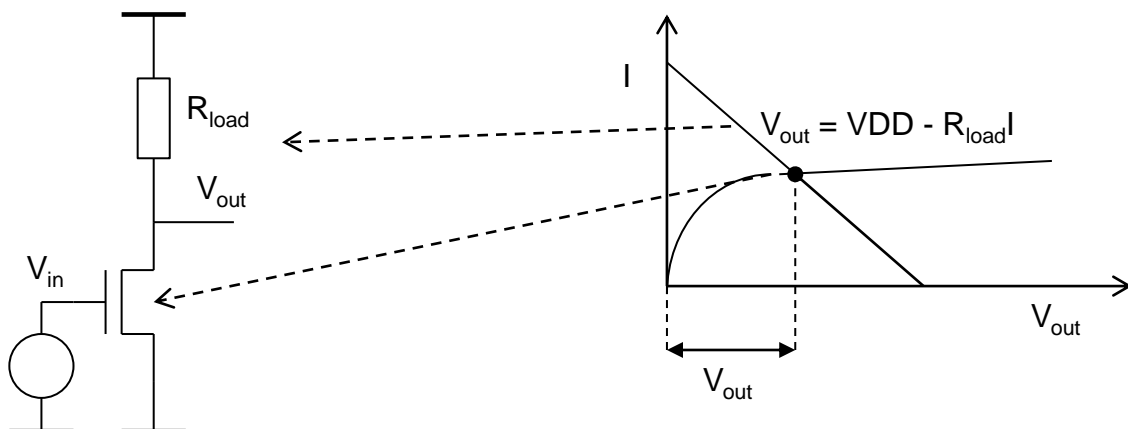

Fig 9: Amplifier realized with transistor and resistor

Graphical analysis

We draw the I_{ds} - V_{ds} characteristics of the transistor and the I-V characteristic of the resistor in the same graph, as shown in Fig 9. The output voltage corresponds to the intersection point of the transistor and resistor characteristics. If the input voltage increases, the transistor characteristic moves upwards and the output voltage moves from the positive supply VDD to GND, as shown in Fig 10. This analysis is called the load line analysis.
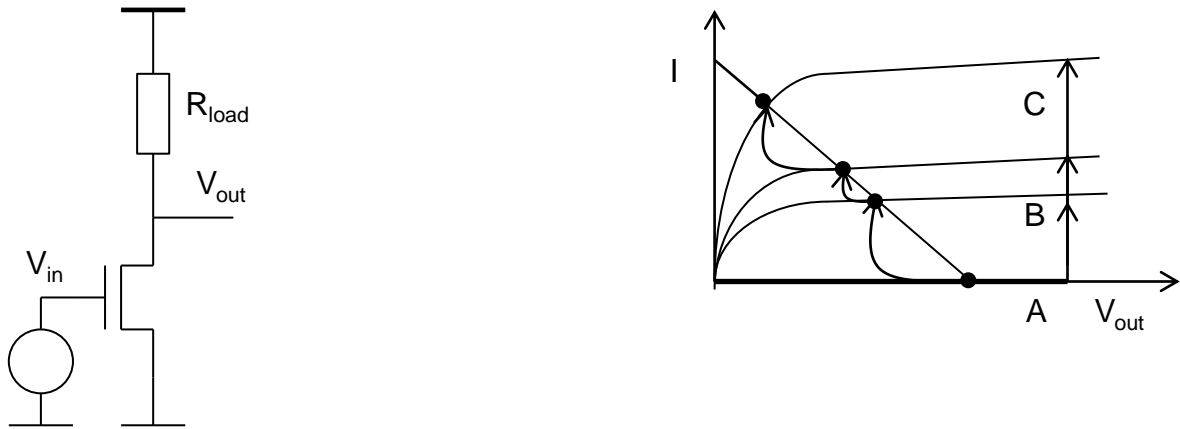
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Fig 10: Graphical DC analysis of the amplifier

We distinguish different V_{in} regions of characteristics (Fig 11) where the input transistor is off (A) ($V_{gs} = V_{in} < V_{th}$), where the transistor is in saturation (B) ($V_{ds} = V_{out} > V_{gs}-V_{th} = V_{dssat}$) and the region (C) where the transistor is in the triode region ($V_{out} < V_{dssat}$). Only in region B, the voltage gain is high. (We neglect in this lecture the subthreshold current and the body effect.)
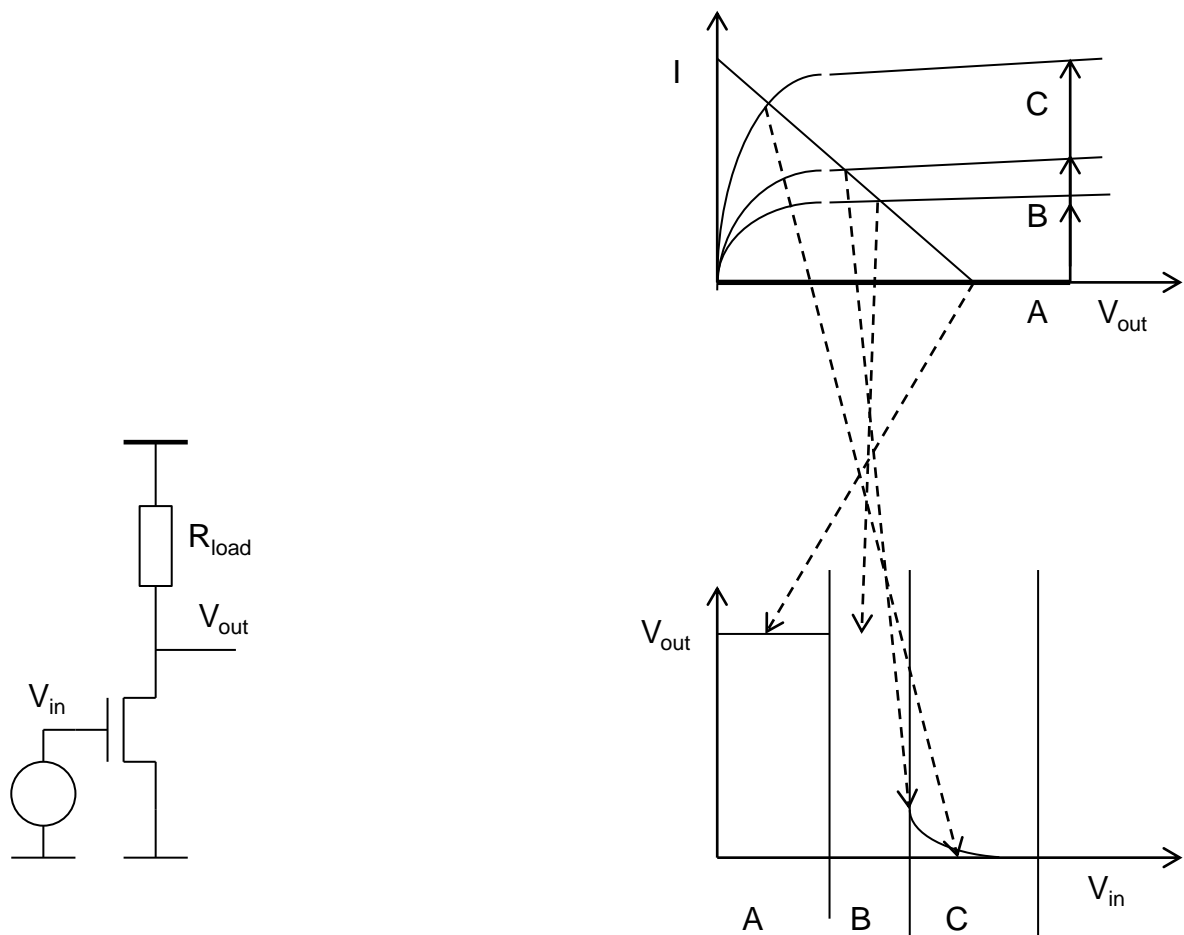

Fig 11: Operating regions of the amplifier

Let us try to use the amplifier, without additional circuits, to amplify signals. We connect a voltage signal source to the input. It would not work properly for several reasons.

The output voltage will probably be constant ~ VDD for all input signals, as Fig 12 shows.
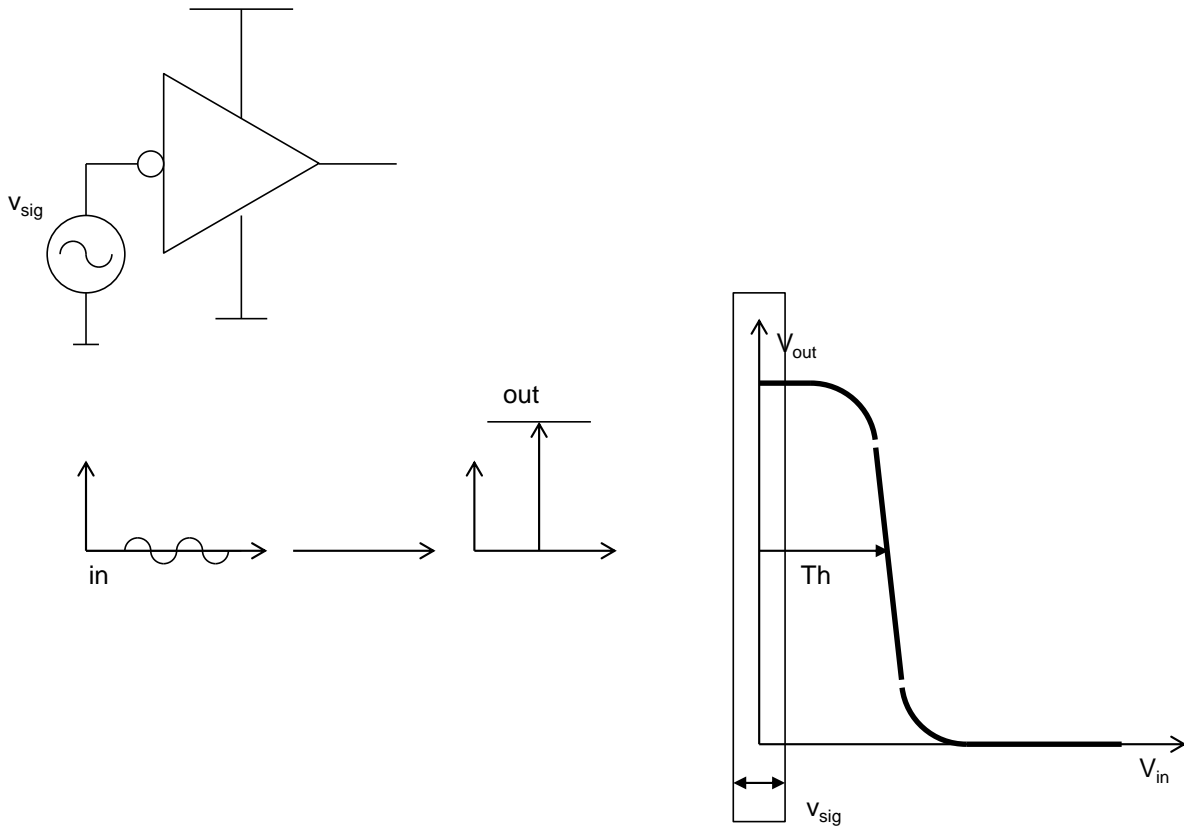

Fig 12: Amplifier without feedback. The input signal is outside of the active region

Only when the signal source is connected to an additional DC voltage source (VDC) and the DC voltage is precisely chosen to match the amplifier threshold Th (Fig 12), a signal is present at the output.
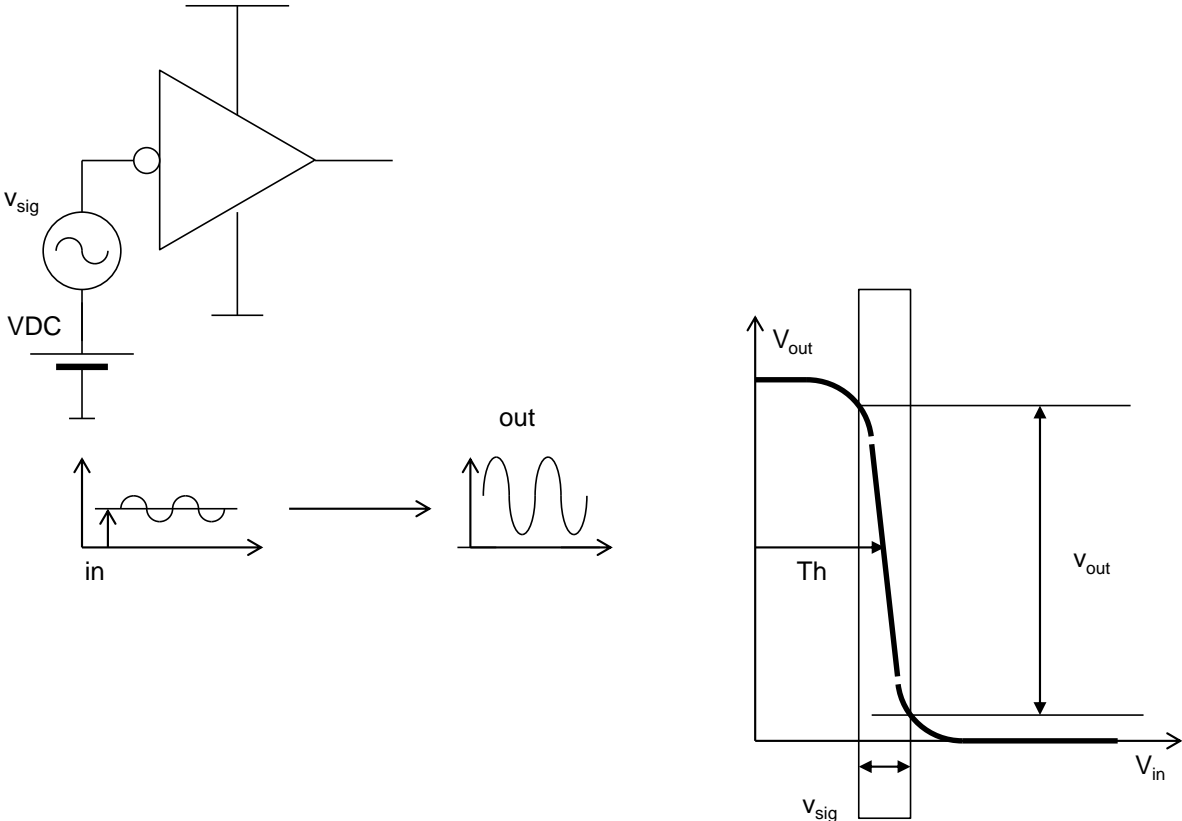
Fig 13: Amplifier without feedback. Bias source VDC assures that the input signal is in the active region

The output signal would probably be distorted and look more like a clock signal as illustrated in Fig 14.
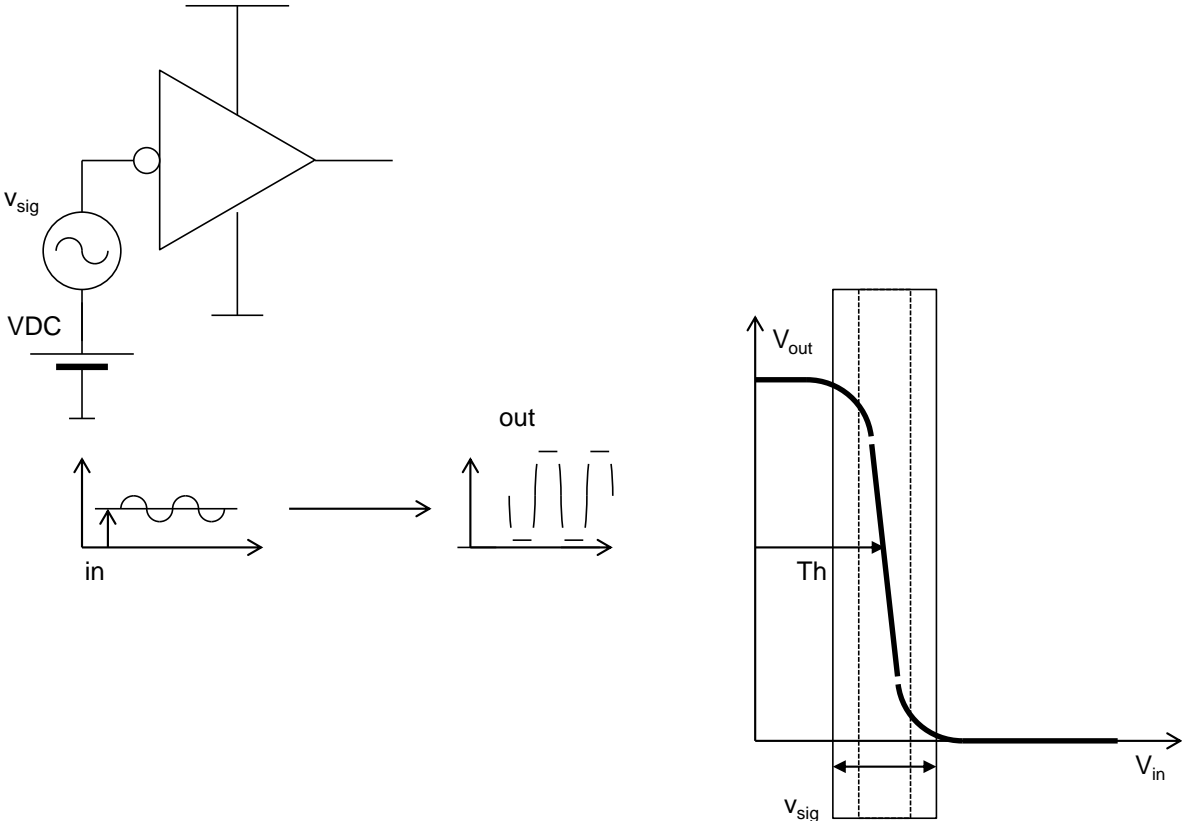
Fig 14: Amplifier without feedback. If the input signal amplitude is large, output signal is distorted

A change in temperature would influence the amplifier. It can cause that the input signal exits the active region with high amplification. The output voltage will become weaker, Fig 15.
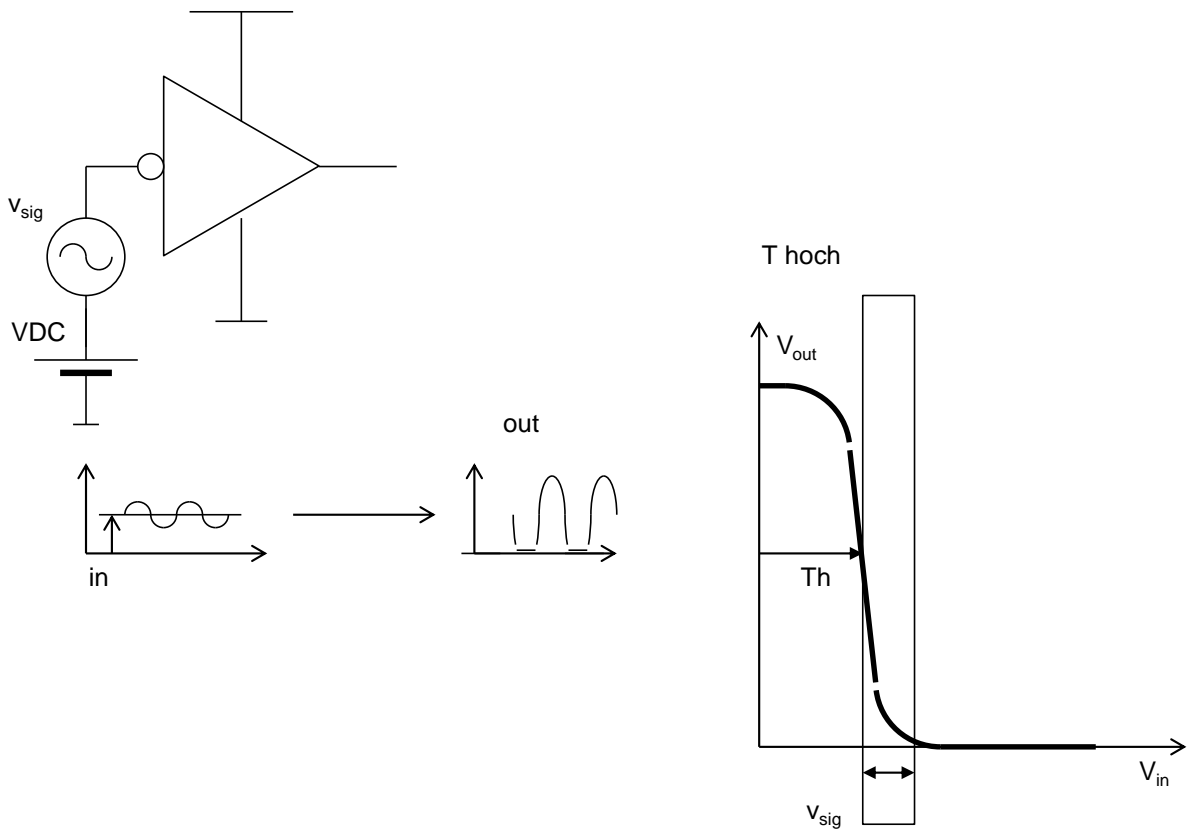
Fig 15: Amplifier without feedback. Temperature influences the amplifier characteristics

Small signal model

We can derive a small signal circuit for the active region of the amplifier, as shown in Fig 16. The amplification (voltage gain) is: $A = v_{out}/v_{in} = -g_m (r_{ds} \| R_{load})$
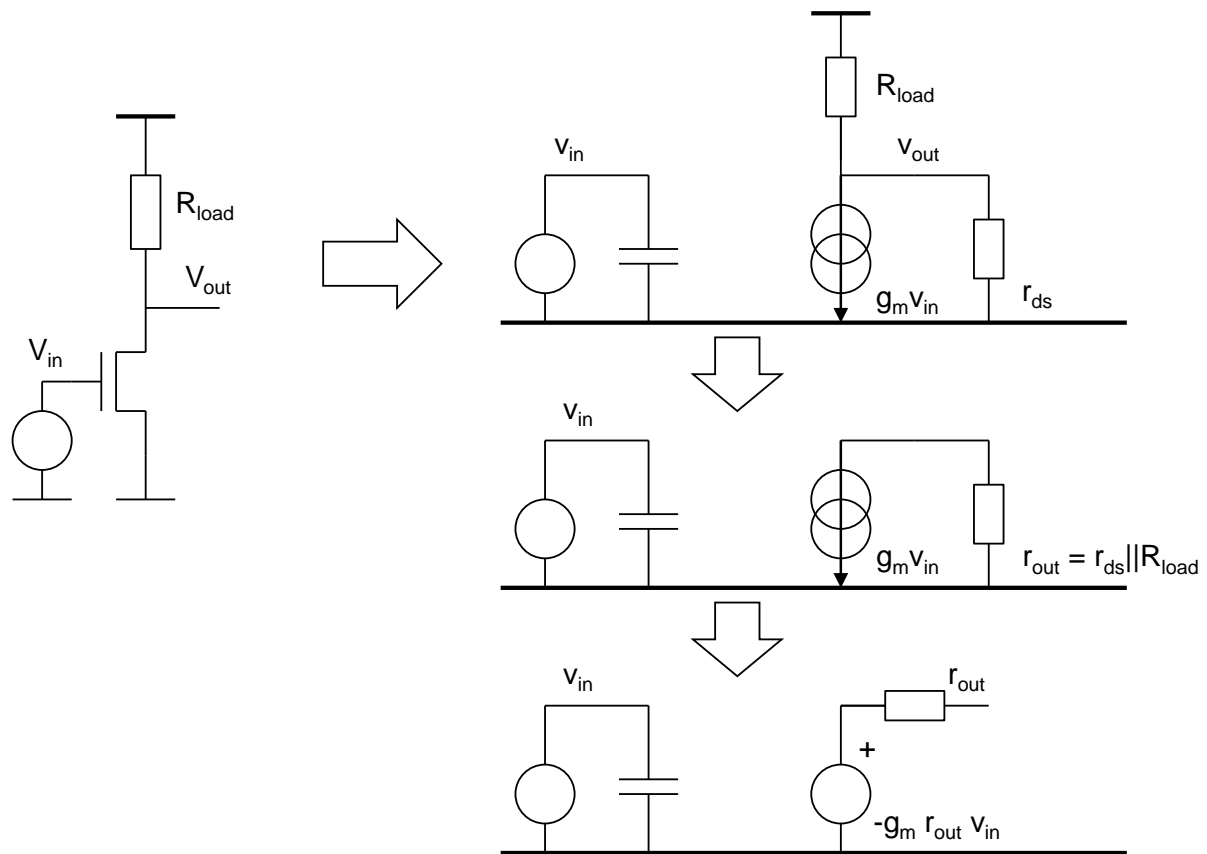
Fig 16: Small signal model of the amplifier

Inverting amplifier

An important technique that makes from nonlinear active components good linear amplifiers is the feedback. The feedback allows the design of precise amplifiers and oscillators.

In this lecture we will present one of the basic circuits with feedback – the inverting amplifier.

Here we will show two versions of an inverting amplifier with feedback:

First variant: amplifier with continuous feedback

In Fig 17 we use a slow and "strong" resistive feedback for DC voltages and a faster and "weaker" capacitive feedback for AC signals. Such an amplifier does not amplify the slow signals; the DC feedback only serves to stabilize the operating point. The AC signals are amplified.

The circuit consists of a resistor R_{fb} and two capacitors C_{in} and C_{fb}. The AC signal voltage source is connected to C_{in}.
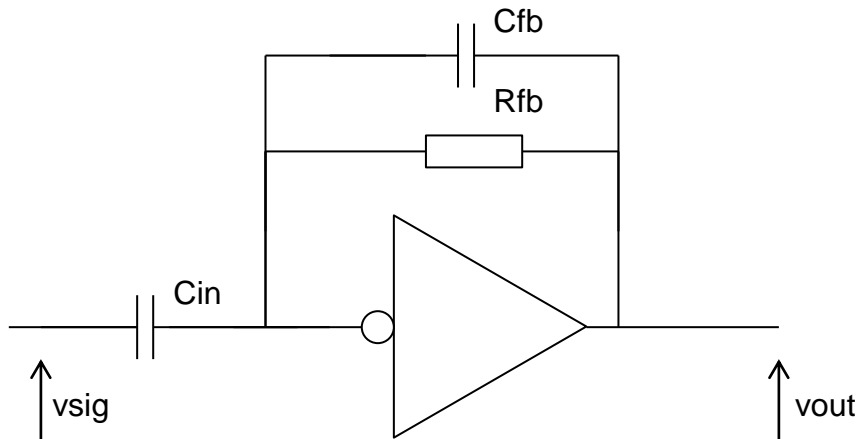
DC and AC analysis are done separately.

Fig 17: Inverting amplifier with continuous feedback

Second variant of the inverting amplifier is shown in Fig 18. It is called the switched amplifier (or switched capacitor amplifier). This amplifier uses a switch instead of the resistor. We use a switch to assure a correct working point.
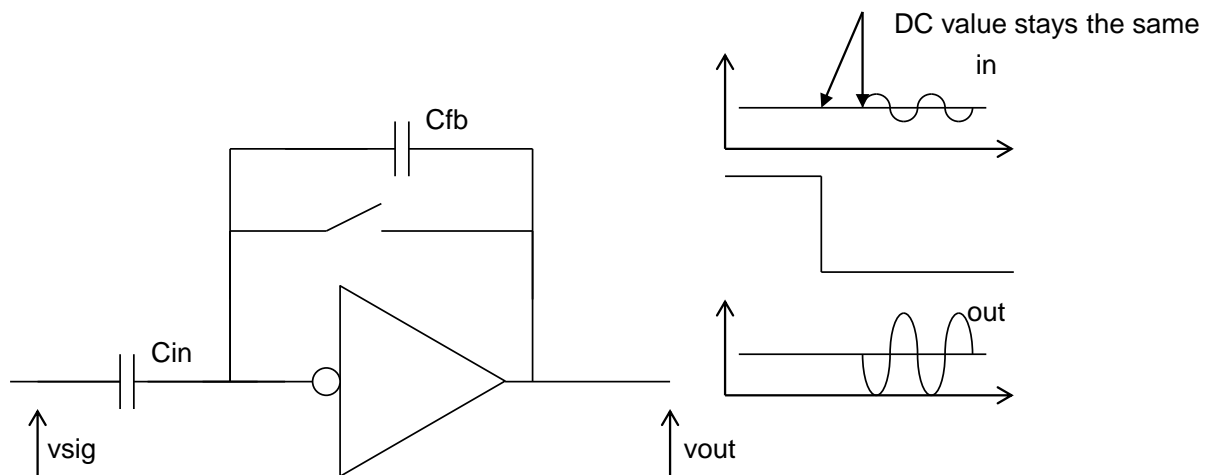

Fig 18: Switched inverting amplifier

Let us now do the DC analysis. The working point should be found. As first we will remove the capacitors, as shown in Fig 19.

The DC circuit consists of the amplifier and the feedback circuit. Note that the amplifier has an infinite input impedance. For this reason no DC current flows through R_{fb} and thus is holds: $V_{out} = V_{in}$.

The DC analysis is the same for both amplifier variants (continuous and switched).

In the case of continuous feedback, R_{fb} represents the resistance of a normal resistor. In the case of the switched amplifier, R_{fb} is the on-resistance of a switch.
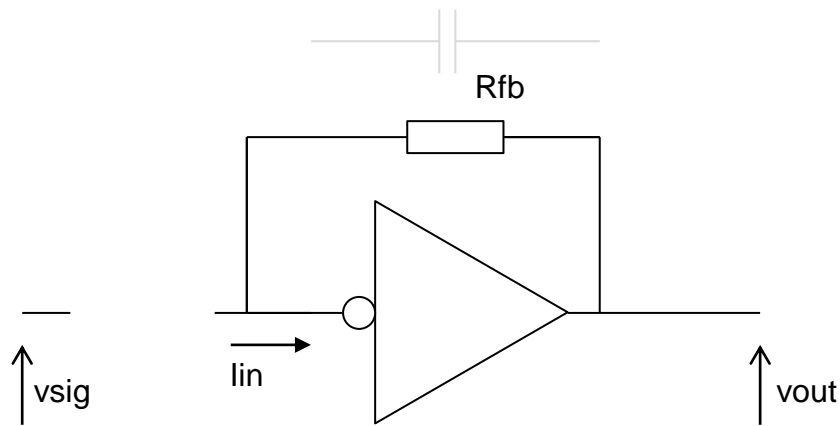
Fig 19: Inverting amplifier DC analysis

We draw the amplifier characteristic and the feedback characteristic in the same graph. The lines intersect in one point, which is the operating point, Fig 20.

It is still to be checked whether the feedback is positive or negative. As a rule, if there is only one intersection point, the feedback is negative. Positive feedback leads usually to several intersections, several stable working points. An example of a circuit with positive feedback is the RAM cell (Fig 21).
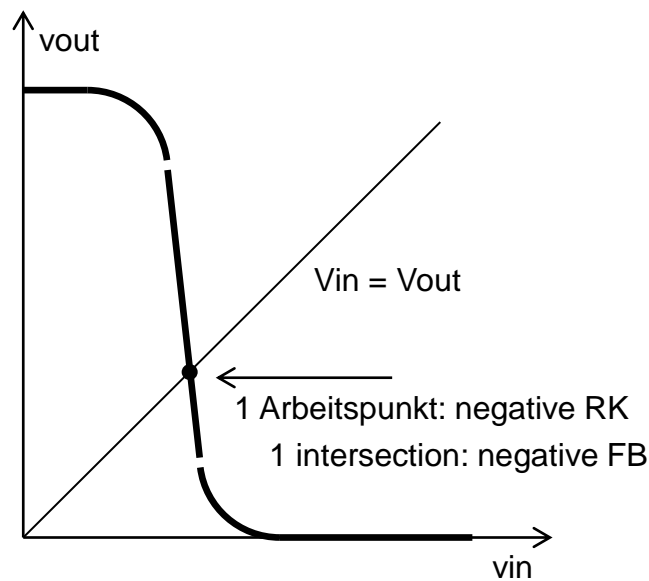

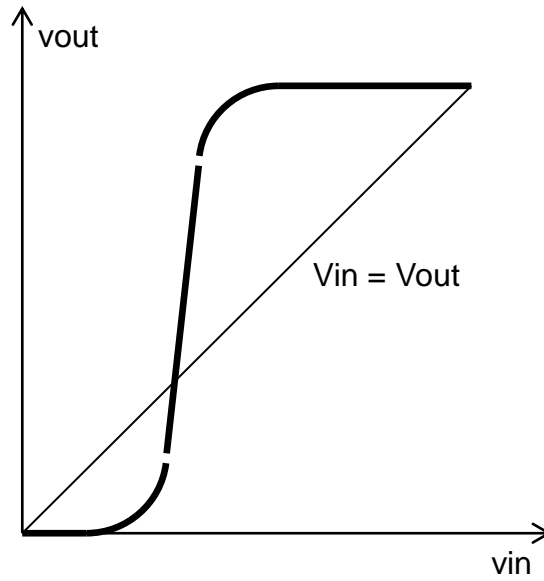*Fig 20: Inverting amplifier DC analysis, graphical solution*

Fig 21: Positive feedback

Fig 22 shows that the working point of our amplifier is in the active region of the characteristics (in the area with high amplification).
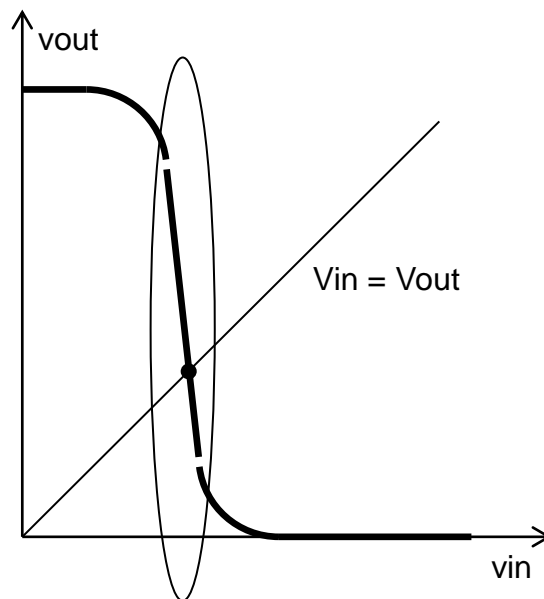

Fig 22: Operating point

Small signal analysis of the inverting amplifier

We can either linearize the input output characteristics of the amplifier or directly use the small signal model from Fig 16. All constant voltage sources are switched off, including VDD.

The following figures illustrate the AC analysis of the inverting amplifier from Fig 17.

We assume that R_{fb} is very large. In this case the impedance of the capacitor C_{fb} ($1/\omega C_{fb}$) much smaller than R_{fb} for all frequencies of interest. Since C_{fb} and R_{fb} form a parallel circuit, we can neglect R_{fb}. The simplified small signal circuit is shown in Fig 23.
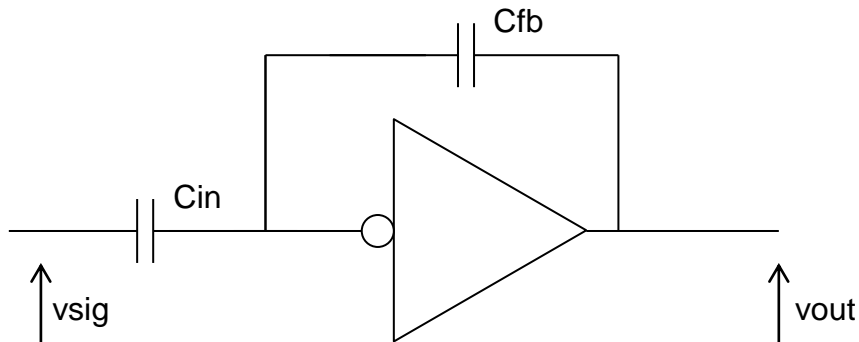

Fig 23: Small signal analysis of the inverting amplifier

In the case of the switched amplifier from Fig 18, we assume that the signals are generated after the switch is opened. Therefore R_{fb} can by also neglected in the AC analysis.

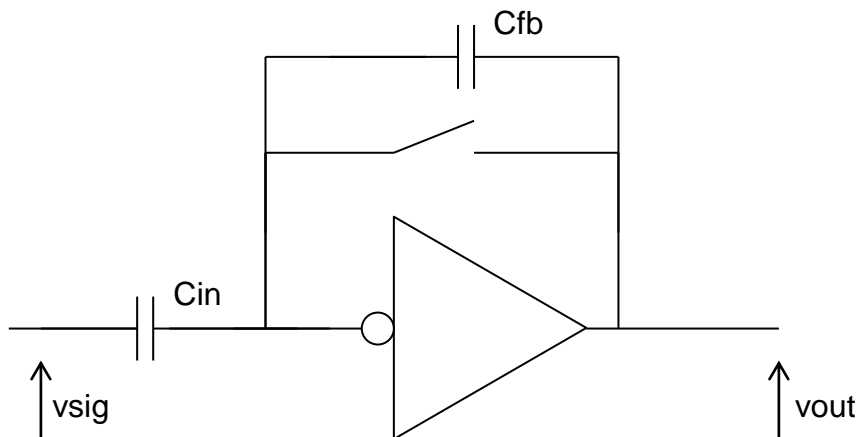The following analysis is valid for both cases, the continuous and the switched amplifier.

Fig 24: Switched amplifier

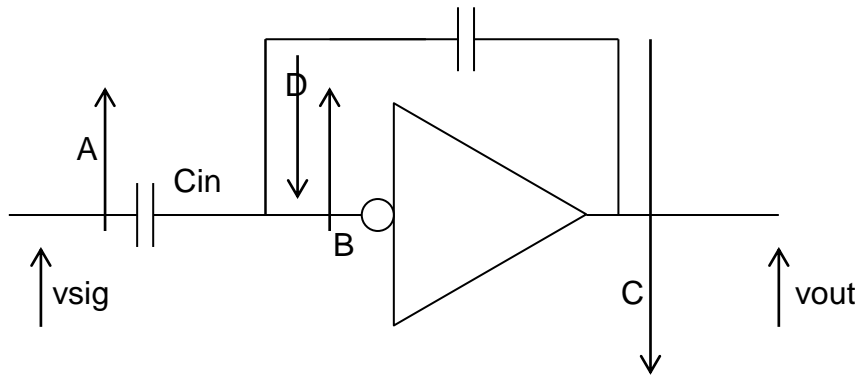The feedback is negative, as illustrated in Fig 25.

Fig 25: If we increase voltage A, B is increased too. C decreases, as well as D. Therefore D acts against input signal. It is negative feedback.

The feedback circuit in our amplifier works in the following way: The voltage from the amplifier output is converted to feedback-current by capacitor C_{fb}. The feedback current is subtracted from the signal current. This type of feedback is called voltage-current feedback. The passive network at the input of the amplifier is called adder, it adds the input and feedback signal, Fig 26.
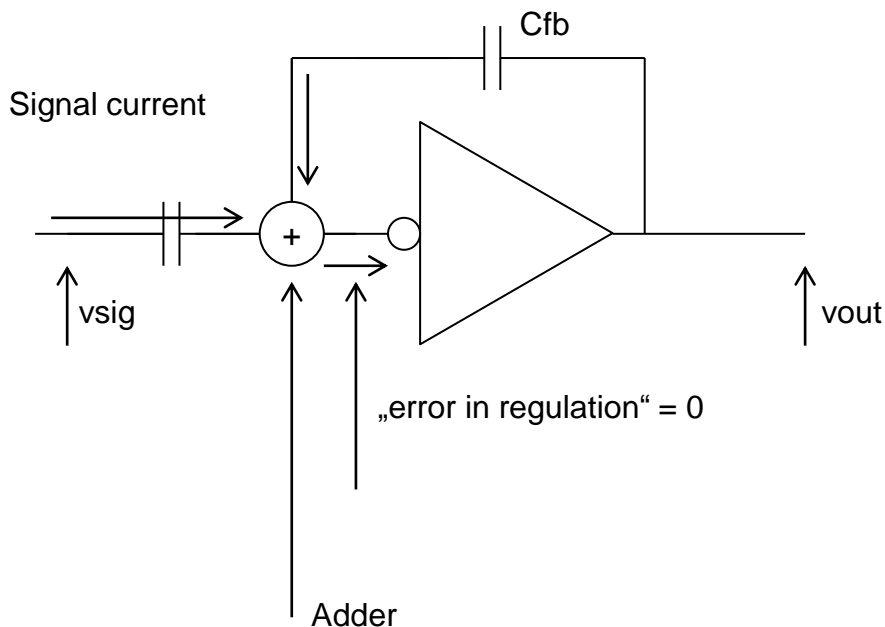

Fig 26: Illustration of voltage-current feedback

Let us calculate the amplification: v_{out}/v_{sig}.

First possibility: We could calculate the currents and voltages using e. g. node potential method, illustrated in Fig 27.

The equations are relatively easy to write. In our circuit, we have only two independent potentials, v_{in} and v_{out} as shown in Fig 27. There is a third node (node "X"), but its voltage is dependent on v_{in}. It is relatively tedious to solve such equations. Long expressions arise which

can only be abbreviated at the very end. It would be even more difficult if R_{fb} was taken into account.
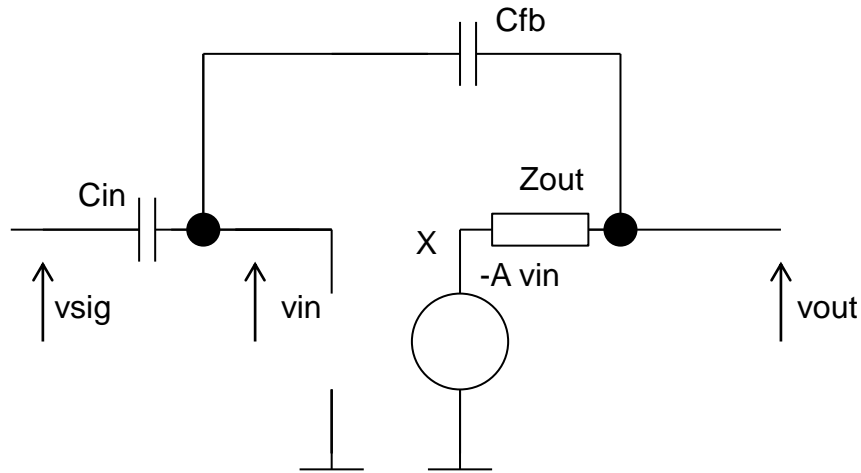

Fig 27: Method of node potentials

For these reasons we use an alternative method for calculating the feedback circuits.

The following formula, known as Mason gain formula, can be used:

$$A_{FB} = \frac{A_{IN}A_{OL}}{1-\beta A} \quad (1)$$

A_{fb} is the gain of a circuit with feedback.

For the calculation of factors A_{in}, A_{ol}, FF and βA, the feedback loop should be cut in a suitable point, normally at the input of the amplifier. This is shown in Fig 28.
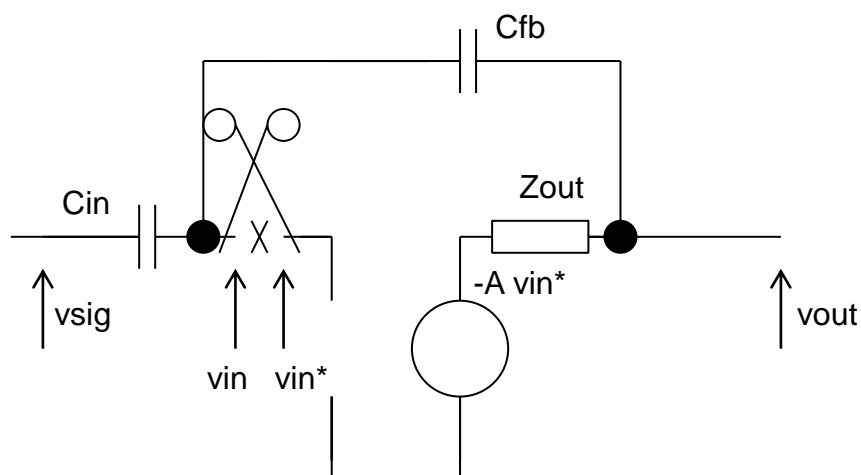

Fig 28: The feedback is cut at the input of the amplifier

A_{in} is the amplification in the "feedback-adder". It is measured from the signal source (the main input) to the input of the amplifier, as Fig 29 shows.
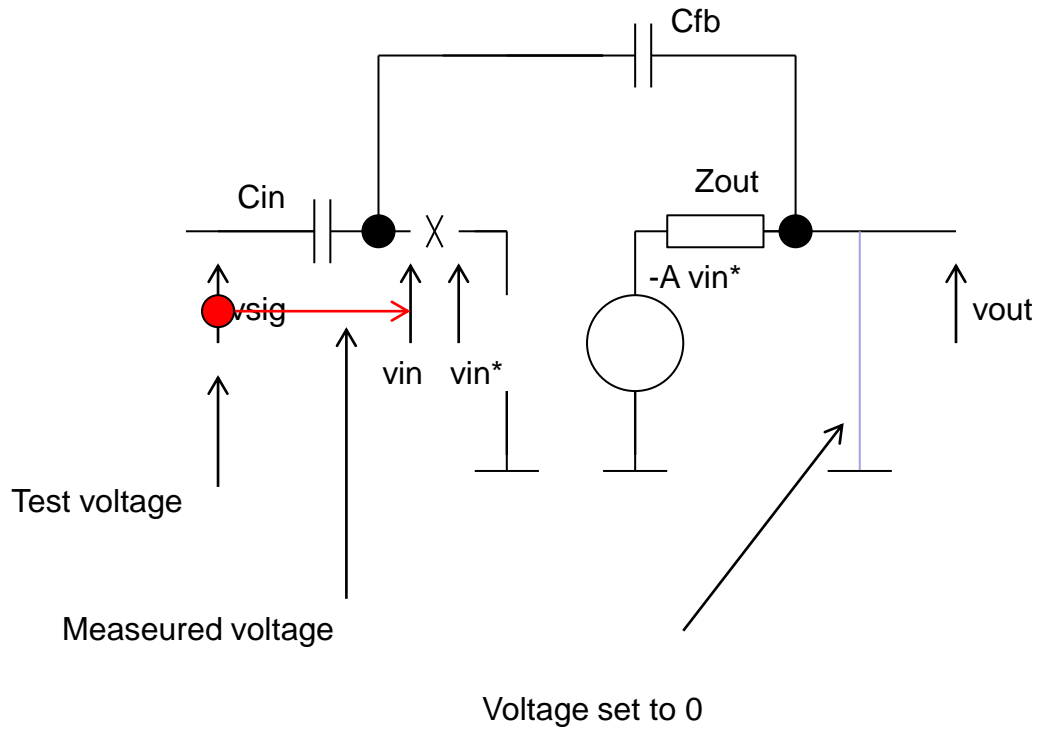
Fig 29: Input gain

A_{ol} is the open loop gain. It is measured from the amplifier input to the main output, as Fig 30 shows.
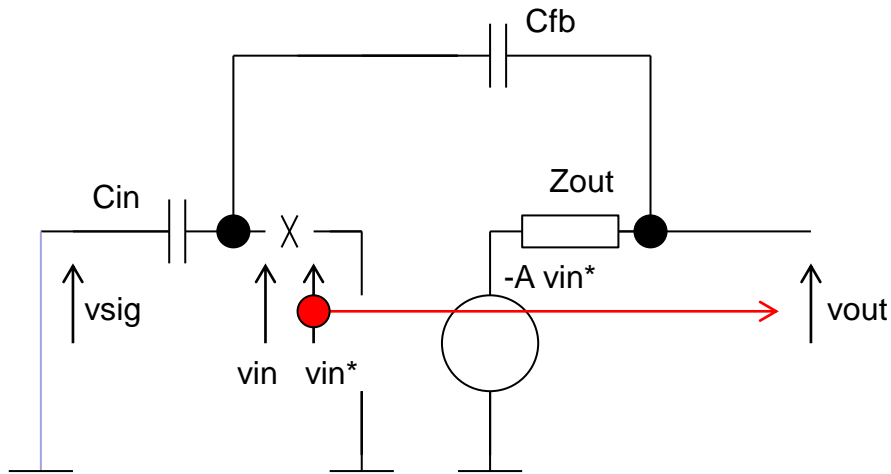

Fig 30: Open loop gain

FF is the feed forward, it describes the signal transmission from the main input to the main output through the feedback network when the amplifier is switched off. FF is usually negligible.
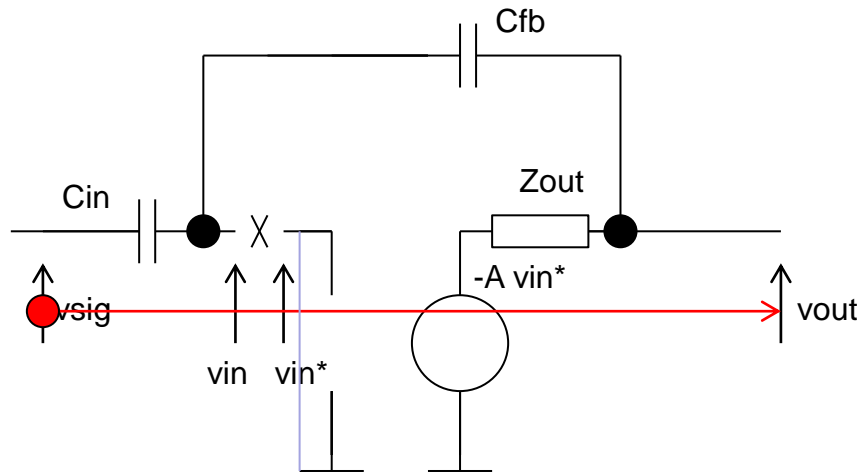
Fig 31: Feed forward

BA is the loop gain. It is the amplification from the input of the amplifier to the point before the intersection point, as shown in Fig 32. In case of a negative feedback βA is negative (more precisely: DC part of βA is negative).
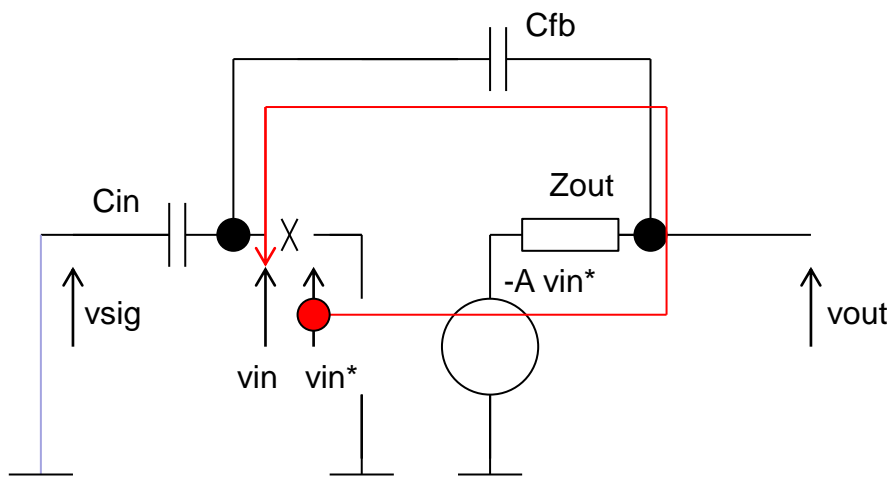

Fig 32: Loop gain

It holds $\beta A = \beta \times A_{ol}$.

β is the feedback from the main output to the amplifier input, Fig 33.

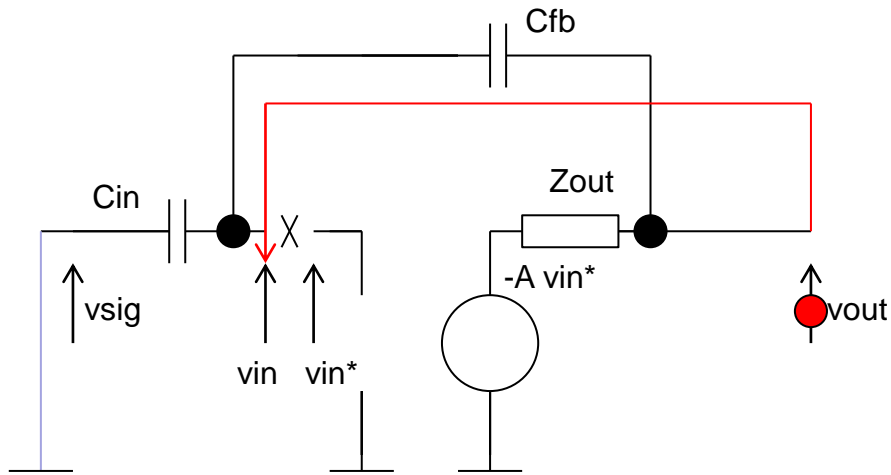Now Let us calculate the individual terms. We use four simplified schematics.

Fig 33: Feedback

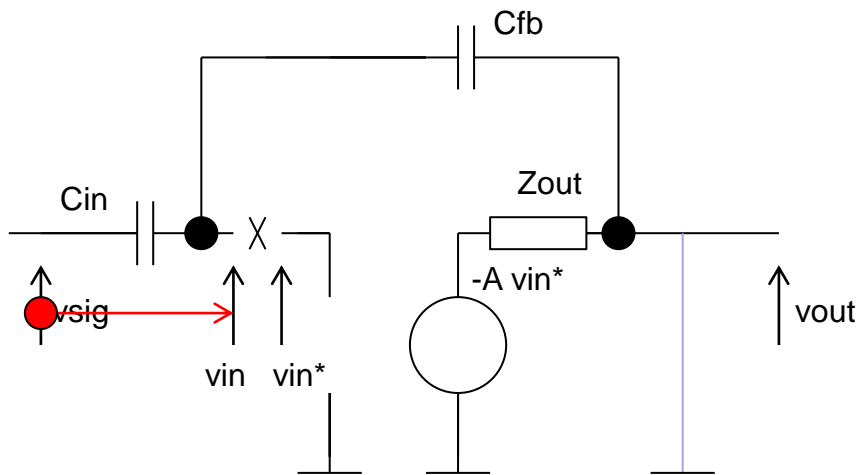Let us first calculate A_{in} (Fig 34).

Fig 34: Calculation of A_{in}

A_{in} is defined as v_{in}/v_{sig}.

The following is important: the main output has to be short-circuited when calculating the A_{in}. (In our case, the output is a voltage. If the output was a current, we would have to cut (to open) the output line.)

The circuit becomes simpler, it is a voltage divider (Fig 34).

$$v_{in} = \frac{Z_{FB}}{Z_{FB} + Z_{in}} v_s$$

$$A_{in} = \frac{Z_{FB}}{Z_{FB} + Z_{in}}$$

Let us calculate A_{ol} (Fig 35). Aol is defined as a $v_{out} / v_{in}*$. Between $v_{in}*$ and v_{out} there is the amplifier with a voltage divider at the output (Fig 35).
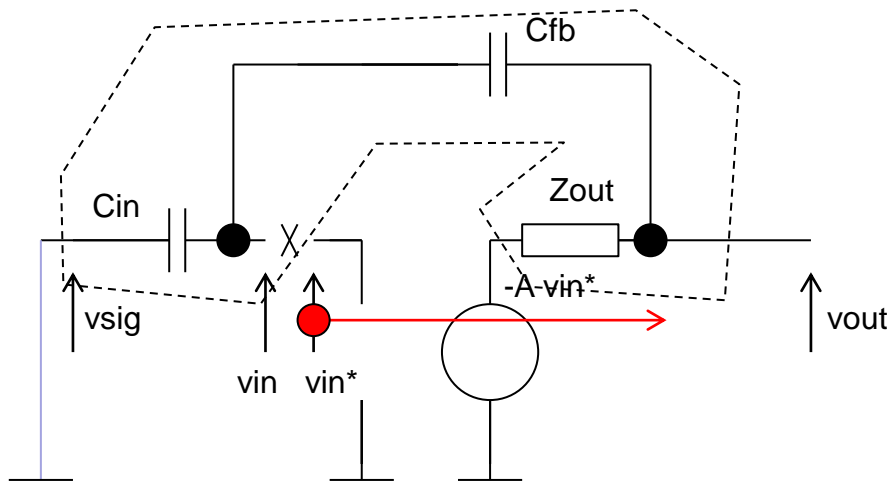
Fig 35: Calculation of A_{ol}

When we assume $R_{out} = 0$, we obtain $A_{ol} = - A$. Otherwise it would be

$A_{ol} = -A \, (z_{in} + z_{fb}) \, / \, (z_{in} + z_{fb} + z_{out})$.

The advantage of the method is that it is possible to simplify the factors from the beginning. The formula for voltage dividers is often used.
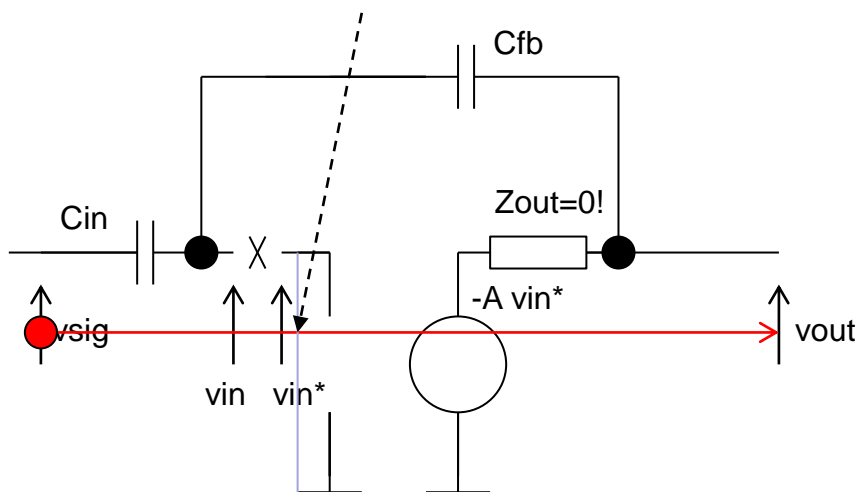

Fig 36: Calculation of FF

Let us calculate FF. For this calculation the amplifier must be switched off. This is done by shorting the input voltage $v_{in}*$. The circuit gets simpler as shown in Fig 37. FF is defined as: v_{out}/v_{signal}. We calculate $FF = z_{out}/(z_{in} + z_{fb} + z_{out})$. For $R_{out} \sim 0$, $FF \sim 0$.
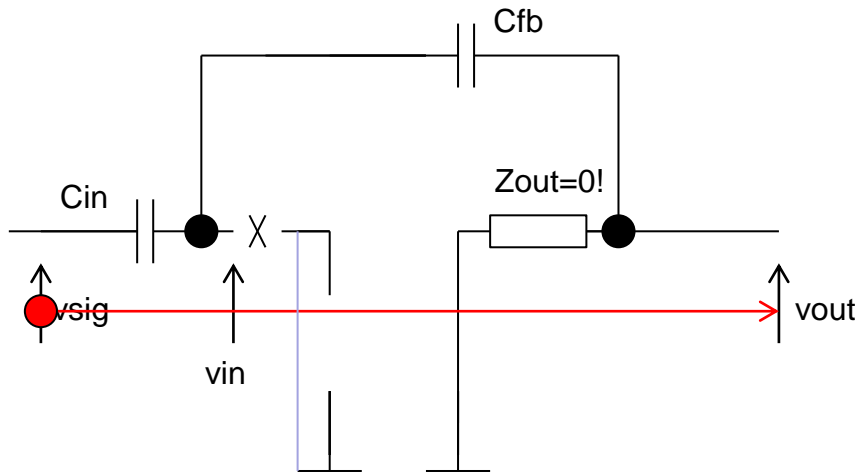
Fig 37: Calculation of FF (2)

Finally, the loop gain is calculated, Fig 38. BA is defined as v_{in} / v_{in}^*.
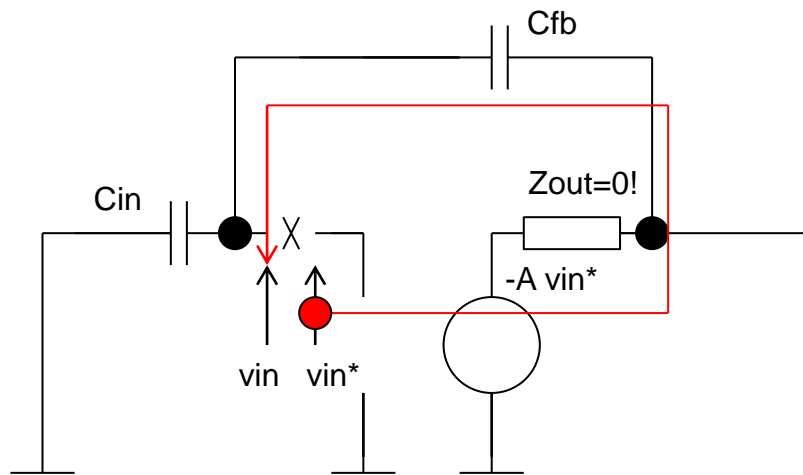

Fig 38: Calculation of loop gain

The result is:

$$\beta A = -A \frac{Z_{in}}{Z_{in} + Z_{FB} + Z_{out}}$$

for $Z_{out} = 0$ it is:

$$\beta A = -A \frac{Z_{in}}{Z_{in} + Z_{FB}}$$

Finally, if we substitute the values of the terms in (1), we obtain the gain with feedback:

$$A_{FB} = \frac{-A \frac{Z_{FB}}{Z_{FB} + Z_{in}}}{1 + A \frac{Z_{in}}{Z_{FB} + Z_{in}}}$$

or

$$A_{FB} = \frac{-A \dfrac{C_{in}}{C_{FB} + C_{in}}}{1 + A \dfrac{C_{FB}}{C_{FB} + C_{in}}}$$

A common assumption is $|\beta A| \gg 1$. It holds if the active amplifier has a large amplification. In this case:

$$A_{FB} = -\frac{A_{in} A_{ol}}{\beta A_{ol}} = -\frac{A_{in}}{\beta}$$

or

$$A_{FB} = -\frac{C_{in}}{C_{fb}}$$

It is interesting (and practical) that the amplification A_{fb} does not depend on A. Open loop gain A is difficult to control and temperature dependent. On the other hand, the passive components and especially the capacitors have stable and well-known properties. As the A_{fb} depends only on capacitances, good accuracy and stability can be expected. Even better, we can realize the two capacitors in the same way, e.g. as metal-metal capacitors and thus further increase the accuracy. The gain with feedback is reduced by βA compared to the value without feedback. The amplifier is "sacrificed" to stabilize the circuit.

Virtual mass

The voltage at the input of the amplifier is practically 0, which follows from the formula

$v_{in} = v_{out} / A = -v_{sig} \, C_{in}/(C_{fb} \, A)$.

In other words, there is no (AC) voltage at the amplifier input. The input is a virtual mass. In well-built systems with feedback, the error term is ~ 0.

Notice that we could have assumed $v_{in} = 0$ at the beginning, in order to simplify the circuit analysis. It is easy to obtain $v_{out} = -Z_{fb}/Z_{in} \, v_{sig}$ directly if $v_{in} = 0$ is assumed. The voltages form a kind of lever as shown in Fig 39.

However, in the case of a simplified analysis (assuming $v_{in} = 0$), we would not know which condition A should meet, or how large it must be.
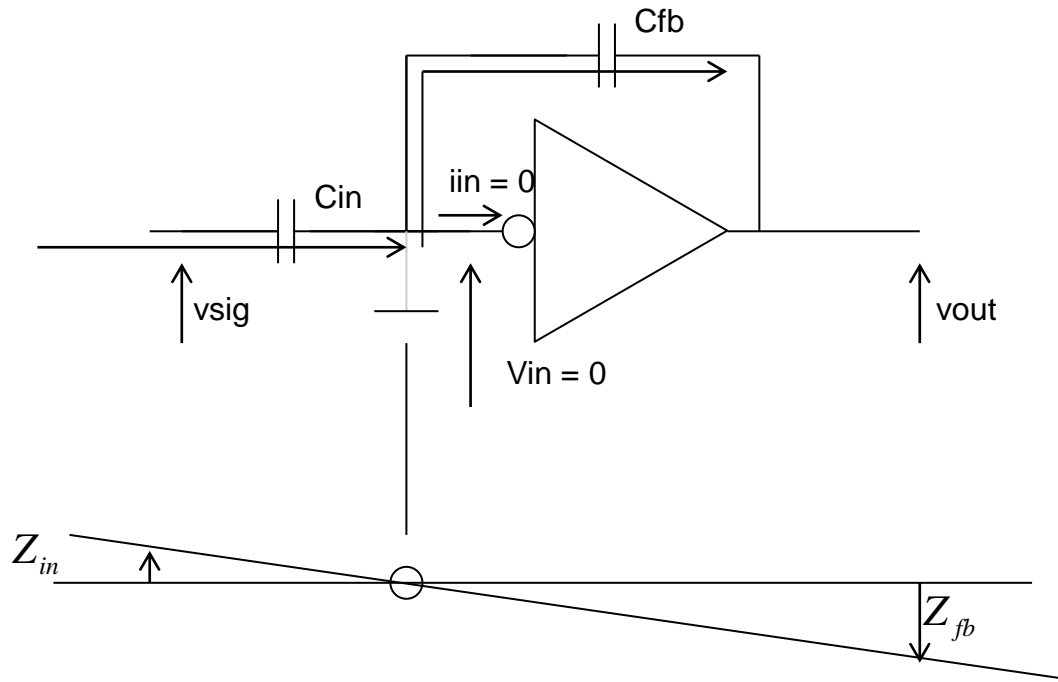
Fig 39: Virtual mass

Dimensioning von A

Let us consider again at the condition $\beta A \gg 1$ i.e. $A\, C_{fb} / (C_{in} + C_{fb}) \gg 1$. Suppose we also want a relatively high amplification with feedback, for example $A_{fb} = 10$. We can achieve it by choosing: $C_{in} = 10\, C_{fb}$. The condition $\beta A \gg 1$ can be rewritten as $A\, C_{fb}/C_{in} = A/A_{fb} \gg 1$. From the formula for A_{fb} one can derive the following condition: If we want a certain amplification with feedback (e.g. $A_{fb} = 10$), the active amplification must be at least 1 to 2 in orders of magnitude (e.g. $A = 1000$). The value of A depends on the desired resolution: e.g. if $A \sim 10\, A_{fb}$, A_{fb} is about 10% lower than desired. $A \sim 100\, A_{fb}$, A_{fb} is 1% lower, etc.

We have a trade-off between accuracy (βA large) and amplification (β small).